Computer Assisted Mechanics and Engineering Sciences, 14: 569–578, 2007.

Evolutionary multi-objective optimization of hybrid laminates

Witold Beluch[1)], Tadeusz Burczyński[1,2)], Adam Długosz[1)]
[1)] *Silesian University of Technology,*
Department for Strength of Materials and Computational Mechanics
Konarskiego 18A, 44-100 Gliwice, Poland
[2)] *Cracow University of Technology, Institute of Computer Modelling*
Warszawska 24, 31-155 Cracow, Poland

(Received in the final form September 18, 2007)

The aim of the paper is to prepare an efficient method of the optimization of the hybrid fibre-reinforced laminates. Since the several optimization criteria which cannot be satisfied simultaneously are proposed, the multi-objective optimization methods have been employed. Different optimization criteria connected with the laminates' cost, the modal properties and the stiffness are considered. The multi-objective evolutionary algorithm which uses the Pareto approach has been used as the optimization method. To solve the boundary-value problem the finite element method commercial software has been employed. Numerical examples presenting the effectiveness of the proposed method are attached.

Keywords: multi-objective optimization, evolutionary algorithm, multi-layered laminate, modal analysis

1. INTRODUCTION

Composite is a material made of two or more permanently joined materials on the macroscopic level [16]. They usually consists of two phases: i) the matrix, being the binder, and ii) the reinforcement playing the role of main bearing elements. Polymer matrices with carbon, graphite, glass, boron or aramid fibers are the most typical ones.

The fibre-reinforced composites — laminates — have especially great strength/weight and elasticity/weight ratio with comparison to conventional, usually isotropic structural materials, like steel or aluminium alloys [15]. Laminates are usually made up of many plies (laminas) with different fibres orientation, while the fibres in particular laminas are placed unidirectionally.

It is possible to "construct" laminate properties by manipulating a few parameters, like: components material, fibres orientation, stacking sequence or layers thicknesses. As a consequence, laminates become more and more frequently used in modern industry as high-efficient materials.

As the cost of laminates increases expeditiously with their strength, laminates can be composed of different materials to reduce the cost ensuring the high efficiency of the laminate [1]. Laminates which plies are made of different materials are called the hybrid ones. There are a few types of hybrid laminates [14]: i) interply hybrids with (at least) two homogeneous reinforcements stacked; ii) intraply hybrids in which tows or constituent fibres are mixed in the same layer; iii) intermingled hybrids with highly randomly mixed fibers of different kinds; iv) selective placement in which additional reinforcement is located in "critical" places; v) superhybrids stacked up of metal foils or metal composite plies in a given sequence and orientation. The first group of hybrid laminates is considered in the present paper. Typically, the internal layers of interply laminates are built of a cheaper material having worse properties while the external layers are composed of a 'better', but more expensive material.

In a classical optimization optimizing a single function leads to one optimum which may be a global one (or not). The single-objective optimization problems for hybrid laminates were successfully solved in previous papers, e.g. [5, 6].

In many practical engineering problems several goals with opposing characteristics must be satisfied simultaneously. It means that decreasing the value of one of the functions may increase the value of another [12]. The common approach in this case is to arbitrarily choose one objective and incorporate the other objectives as constrains. In the present paper the considered problem is solved by means of the Pareto approach. The multi-objective evolutionary algorithm is coupled with finite element method (FEM) commercial software to solve the multi-objective optimization problem for laminates.

2. Laminates' mechanics

Generally, composites are anisotropic materials. Treating fibre-reinforced laminates made of many laminas with uniaxially oriented fibres as orthotropic materials, the number of the independent elastic constants in laminate decreases from 21 to 9 (fully anisotropic material). Taking into account the fact, that the thickness of the laminate is usually small comparing with remaining dimensions, laminate can be treated as 2-dimensional. As a result the Kirchhoff–Love thin plate hypothesis could be applied and the number of independent elastic constants reduces to 4 [11]: axial and transverse Young's module E_1, E_2, axial-transverse shear modulus G_{12} and axial-transverse Poisson ratio ν_{12}.

For the plane-stress state the constitutive equation for a single layer of the laminate (in the in-axis orientation) has the form

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \tag{1}$$

where σ_{ij} – stress vector; ε_{ij} – strain vector; E_1, E_2 – axial and transverse Young's moduli, respectively; G_{12} – axial-transverse shear modulus; ν_{12}, ν_{21} – axial-transverse and transverse-axial Poisson ratios, respectively.

The Poisson ratio ν_{21} depends on other elastic constants in the following way,

$$\nu_{21} = \nu_{12}\frac{E_2}{E_1}\,. \tag{2}$$

The resultant laminate forces **N** and moments **M** referred to the unit cross-section width of the laminate satisfy the matrix equation,

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \varepsilon^o \\ \kappa^o \end{bmatrix}, \tag{3}$$

where $\mathbf{A} = [A_{ij}]$, $\mathbf{B} = [B_{ij}]$, $\mathbf{D} = [D_{ij}]$ – in-plane, coupling and out-of-plane stiffness matrices; ε^o – strains at the mid-plane; κ^o – curvatures at the mid-plane.

If the layers are arranged symmetrically to the mid-plane, the laminate is called symmetrical. In such laminates **B** is a null matrix ($B_{ij} = 0$) and shield and bending states are uncoupled. The **A** matrix fully describes the shield state while the matrix **D** fully describes the bending state. The symmetrical laminates are considered in present paper.

The dynamical properties of structures can be determined by means of modal analysis. The modal analysis can be performed theoretically or experimentally. The theoretical modal analysis is usually based on the numerical methods and should be experimentally verified on real structures. Two main goals can be achieved by means of modal analysis: i) the diagnostics of the structures; ii) the optimization of the structures [17].

4. The eigenfrequency problem for a rectangular hybrid laminate plate of length a, width b and thickness h in directions x, y and z, respectively, can be presented as [2]

$$\rho h\omega^2 w = D_{11}w_{,xxxx} + 4D_{16}w_{,xxxy} + 2(D_{12} + 2D_{66})w_{,xxyy} + 4D_{26}w_{,xyyy} + D_{22}w_{,yyyy} \tag{4}$$

where ω – eigenvalue vector; w – deflection in the z direction, D_{ij} – bending stiffness, ρ – mass density.

3. THE FORMULATION OF THE OPTIMIZATION TASK

3.1. The multi-objective optimization

The objective of the optimization task is to find the optimal set of ply angles and the number of external plies for given criteria related to the eigenfrequencies, cost of the laminate and its stiffness. It is assumed, that the criteria are (or can be) contradictory, so the multi-objective optimization (MOO) methods are used. In the multi-objective optimization solution of the problem is represented by more than one objective function. As all of the objective functions cannot be simultaneously improved, the optimization looks for the solution with acceptable to the designer values of all objective functions. This attitude leads to a set of optimal solutions instead of one.

A MOO problem can be expressed as searching for the vector $\mathbf{x} \in \mathbf{D}$, where $\mathbf{D}$ is a set of admissible solutions being a subset of design space $\mathbf{X}$,

$$\mathbf{x} = [x_1,\ x_2,\ \dots\ x_n]^T, \tag{5}$$

which minimizes the vector of k objective functions

$$f(\mathbf{x}) = [f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ \dots\ ,f_k(\mathbf{x})]^T. \tag{6}$$

It is required for the vector $\mathbf{x}$ to satisfy the m inequality constraints

$$g_i(\mathbf{x}) \geq 0, \qquad i = 1, 2, \dots, m\,, \tag{7}$$

and the p equality constraints

$$h_i(\mathbf{x}) = 0, \qquad i = 1, 2, \dots, p\,. \tag{8}$$

The multi-objective optimization is performed by using the Pareto optimality concept [3]. According to that concept a point $\mathbf{x}^* \in \mathbf{X}$, is Pareto-optimal in the minimization problems if and only if there does not exist another point, $\mathbf{x} \in \mathbf{X}$ such that $\mathbf{F}(\mathbf{x}) \leq \mathbf{F}(\mathbf{x})^*$, with at least one $F_i(\mathbf{x}) < F_i(\mathbf{x}^*)$.

In other words the point is Pareto-optimal if there does not exist another point that improves at least one objective function without deteriorating any other function. A solid line in Fig. 1 represents the set of Pareto optimal solutions, which is called the Pareto front, for MOO problem with two objective functions. The Pareto front is a set of so called non-dominated (or efficient) solutions.

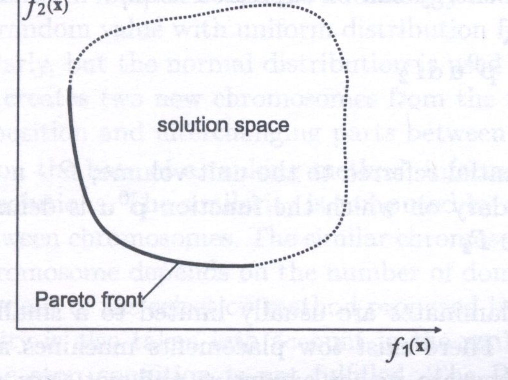

Fig. 1. The Pareto front for the exemplary bi-objective problem

3.2. Optimization criteria

The symmetric, fibre-reinforced hybrid laminates made of two materials are examined. The laminas' number and ply thicknesses of the hybrid laminate are assumed constant. The plies orientations (angles) and the number of external plies of the hybrid laminate are the design variables. As the symmetrical laminates are considered, the number of design variables is a half of the plies number plus one (representing the number of external plies).

One of the reasons of using hybrid instead of simple laminates is the cost reduction of the whole laminate and the balance between the cost and other properties, like strength, modal properties or stiffness.

The set of objective functionals for multi-objective optimization of laminates can be defined as:

- The minimization the cost of the structure.

  As it is assumed that the thicknesses of laminas h_i, the number of plies N and areas of the plate A_i are fixed, the cost of optimized laminate depends only on the number of external, "better" plies and due to this fact it is discrete.

  The cost C of a laminate in the considered case can be calculated as

$$C = [n_e c_e + (N - n_e) c_i]\, h_i A_i \tag{9}$$

  where n_e – the number of external plies; c_e – the unit cost of the external plies material; c_i – the unit cost of the internal plies material.

- The criterion connected with the modal properties of laminate.

  Three optimization criteria connected with the free vibrations of structures are considered:

  1. The maximization of the first eigenfrequency

$$arg\ \max\{\omega_1(\mathbf{x});\ \mathbf{x} \in \mathbf{D}\}. \tag{10}$$

  2. The maximization of the distance between two consecutive eigenfrequencies

$$arg\ \max\{\omega_i(\mathbf{x}) - \omega_{i-1}(\mathbf{x});\ \mathbf{x} \in \mathbf{D}\}. \tag{11}$$

  3. The maximization of the minimum distance between the external excitation frequency ω_{ex} and the eigenfrequency ω_i

$$arg\ \max\{\min(|\omega_i(\mathbf{x}) - \omega_{ex}(\mathbf{x})|);\ \mathbf{x} \in \mathbf{D}\}. \tag{12}$$

- The maximization of the total stiffness of the laminate structure.

  The total potential energy Π_u of a structure can be treated as a measure of the mean stiffness of it. The total potential energy can be expressed as [8]

$$\Pi_u = \int_{\Omega} U(\varepsilon)\, d\Omega - \int_{\Gamma_2} \mathbf{p}^0 \mathbf{u}\, d\Gamma_2 \tag{13}$$

  where $U(\varepsilon)$ – strain potential referred to the unit volume; Ω – a domain occupied by the body; Γ_2 – a part of the boundary on which the function $\mathbf{p}^0\mathbf{u}$ is defined; $\mathbf{p}^0$ – tractions on the Γ_2; $\mathbf{u}$ – displacements on the Γ_2.

The ply orientations of laminates are usually limited to a small set of discrete angles due to the manufacturing process. There exist tow placements machines able to produce laminate with arbitrary ply angles, but they are rather expensive and not very popular. In the present paper discrete as well as continuous variants of the optimization tasks are examined.

4. The multi-objective evolutionary algorithm

In order to solve the multi-objective optimization problem the evolutionary algorithm (EA) with the real-value representation has been proposed [4]. EA is especially useful optimization method in two cases: i) if gradient methods fail since the information about the objective function gradient is hard or impossible to obtain; ii) if objective function is multi-modal, which usually leads the gradient methods to the local optima. The only necessary information for the EA to work is the objective (fitness) function value.

Each possible solution is represented by a vector (chromosome) of design variables (genes). The block diagram of the multi-objective evolutionary algorithm (MEA) is shown in Fig. 2.

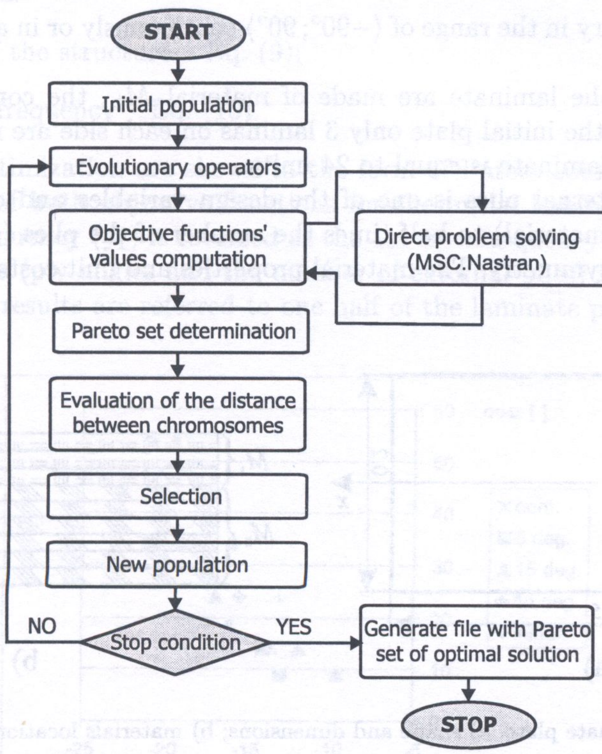

Fig. 2. The block diagram of the multi-objective EA

The proposed multi-objective evolutionary algorithm starts with a population of randomly generated chromosomes. Then, the initial population is modified by evolutionary operators: mutation and crossover. Two kinds of the mutation operators are applied: an uniform mutation and a Gaussian mutation. The operator of the uniform mutation replaces a randomly chosen gene of the chromosome by the new value, which is the random value with uniform distribution from the variable range. The Gaussian mutation works similarly, but the normal distribution is used instead of the uniform one. The simple crossover operator creates two new chromosomes from the two randomly selected ones by cutting them in a random position and interchanging parts between them.

The selection is performed on the base of a ranking method, information about Pareto optimal solutions and the similarity of solutions. The similarity is computed by means of information about (scaled) Euclidean distance between chromosomes. The similar chromosomes have less probability of surviving. The rank of each chromosome depends on the number of dominating chromosomes. This kind of selection scheme is a variant of the selection method proposed by Fonseca and Fleming [10]. The information about similarity is also taken into account in the rank of each chromosome. The procedure is repeated until the stop condition is not fulfilled. The Pareto set is stored in each generation and the collective Pareto set of optimal solutions is eventually generated.

In order to calculate the values of the objective functions the boundary-value problem (direct problem) must be solved. The professional finite element method software MSC.Nastran [13] has been used to solve the boundary-value problem for hybrid laminates.

5. Numerical examples

A symmetric rectangular hybrid laminate plate made up of 18 plies having dimensions presented in Fig. 3a is considered. Each ply has the same thickness equal to $h = 0.0002\,\mathrm{m}$. The initial stacking sequence of the laminate is: (90/15/−45/45/−15/−90/−45/90/45)s. The plate FEM model consists of 200 4-node plane finite elements. The first 5 eigenfrequencies of the plate are considered. They are collected in Table 1.

Each ply angle could vary in the range of $\langle -90°; 90° \rangle$ continuously or in a discrete way, depending on the considered variant.

The external plies of the laminate are made of material M_1, the core plies are made of the material M_2 (Fig. 3b). In the initial plate only 3 laminas on each side are made of material M_1, so the the initial cost of the laminate is equal to 24 units.

The number of the external plies is one of the design variables and can vary from 0 (simple laminate made of weaker material) to half times the number of the plies (simple laminate made of stronger material) due to symmetry. The material properties and unit costs are collected in Table 2.

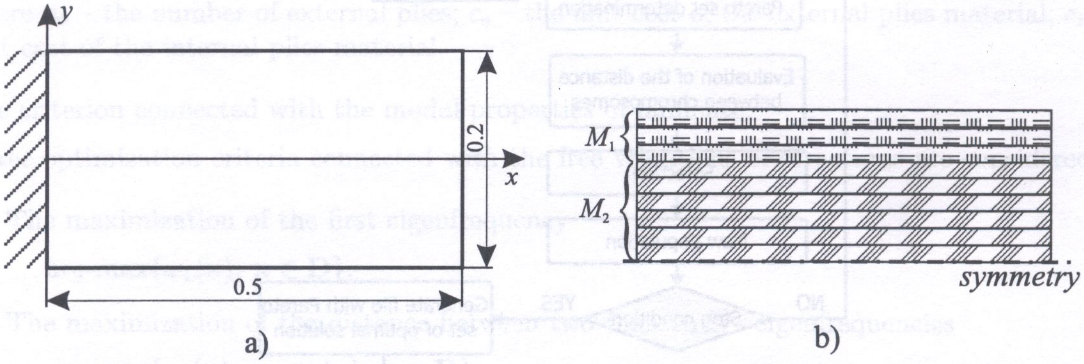

Fig. 3. The laminate plate: a) shape and dimensions; b) materials location for initial plate.

Table 1. The initial laminate plate – the values of the first 5 eigenfrequencies

ω_1 [Hz]	ω_2 [Hz]	ω_3 [Hz]	ω_4 [Hz]	ω_5 [Hz]
13.04	61.27	81.56	196.53	226.80

Table 2. The hybrid laminate – material parameters

Material	E_1 [GPa]	E_2 [GPa]	ν_{12}	G_{12} [GPa]	ρ [kg/m^3]	unit cost [1/(m^3)]
M_1	181	10.3	0.28	7.17	1600	6.0
M_2	38.6	8.27	0.26	4.14	1800	1.0

The parameters of the multi-objective EA are:

– the population size: $p_s = 50$;

– the number of genes in each chromosome: $n_g = 10$;

– the uniform mutation probability: $p_{um} = 0.2$;

– the Gaussian mutation probability: $p_{gm} = 0.1$;

– the simple crossover probability: $p_{sc} = 0.5$;

– stop condition: number of generations.

5.1. Numerical example 1

The aim is to find the optimal number of external plies as well as ply angles in all laminas to satisfy two contradictory criteria:

a) minimize the cost of the structure – Eq. (9);

b) maximize first eigenfrequency – Eq. (10).

The results of the optimization are shown in the form of Pareto solutions in Fig. 4. The point representing the values of both objective functions for the initial laminate is also presented. The scale for the 1st eigenfrequency (f_1) is reversed as the 1st eigenfrequency has been maximized. The values of design variables (ply angles and the number of external plies) for the optimal results are collected in Table 3. All results are referred to one half of the laminate plate due to its symmetry.

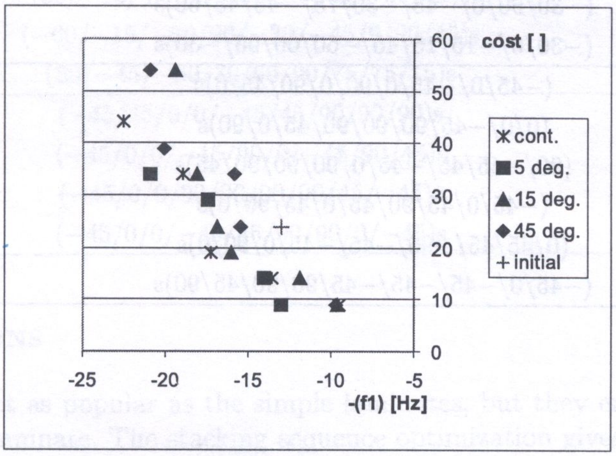

Fig. 4. Numerical example 1 – optimization results

5.2. Numerical example 2

The aim is to find optimal number of external plies and optimum values of ply angles in all laminas to satisfy two contradictory criteria:

a) minimize the cost of the structure – Eq. (9);

b) maximize the gap between 1st and 2nd eigenfrequencies – Eq. (11).

The results of the optimization are shown in the form of Pareto solutions in Fig. 5. The point representing the values of both objective functions for the initial laminate is also presented. The values of design variables (ply angles and the number of external plies) for the optimal results are collected in Table 4. All results are referred to one half of the laminate plate due to its symmetry.

Table 3. Numerical example 1 – design variables values for optimal solutions

case	solution no.	stacking sequence	no of ext. plies	f_1 [Hz]	cost [ ]
initial		(90/15/−45/45/−15/−90/−45/90/45)s	3	13.04	24
cont.	1	(−4.4/10.8/3.2/−0.2/25.9/−72.0/83.1/−56.2/29.2)s	7	22.5327	44
	2	(−35.9/5.8/−0.2/27.1/−88.5/24.4/−60.2/68.1/67.8)s	5	18.9695	34
	3	(11.0/−17.2/9.9/−67.7/0.7/−90.0/−89.9/44.2/−66.7)s	2	17.2344	19
	4	(13.4/−12.6/−84.4/−31.5/33.2/−2.0/62.7/52.1/−80.5)s	1	13.5842	14
	5	(27.78/−48.6/68.3/−79.1/−61.3/−25.3/−29.1/−15.4/72.1)s	0	9.48934	9
5°	1	(0/−15/−25/35/0/90/−65/−65/−20)s	5	20.9221	34
	2	(10/−50/−35/10/−75/75/5/−45/−75)s	4	17.4203	29
	3	(5/−55/−25/45/0/40/−35/60/−85)s	1	14.0206	14
	4	(40/35/−60/−75/−85/50/−80/85/−85)s	0	13.0397	9
15°	1	(−15/90/−30/30/15/−15/90/−60/75)s	9	19.3876	54
	2	(0/90/15/15/−30/−15/45/−15/75)s	5	18.1565	34
	3	(−15/75/−15/30/60/−45/−75/0/45)s	3	16.8979	24
	4	(0/0/90/90/−60/−15/−75/−75/−30)s	2	16.0800	19
	5	(−30/90/0/−45/−30/75/−45/45/60)s	1	11.8603	14
	6	(−30/0/−15/15/45/−60/60/90/−30)s	0	9.57815	9
45°	1	(−45/0/−45/0/90/0/90/45/0)s	9	20.9234	54
	2	(0/0/−45/90/90/90/45/0/90)s	6	20.1023	39
	3	(90/−45/45/−45/0/90/90/90/45)s	5	15.8506	34
	4	(−45/0/45/90/45/0/45/90/0)s	3	15.4900	24
	5	(0/45/45/−90/−45/−45/0/90/0)s	1	13.8400	14
	6	(−45/0/−45/−45/−45/90/90/45/90)s	0	9.7238	9

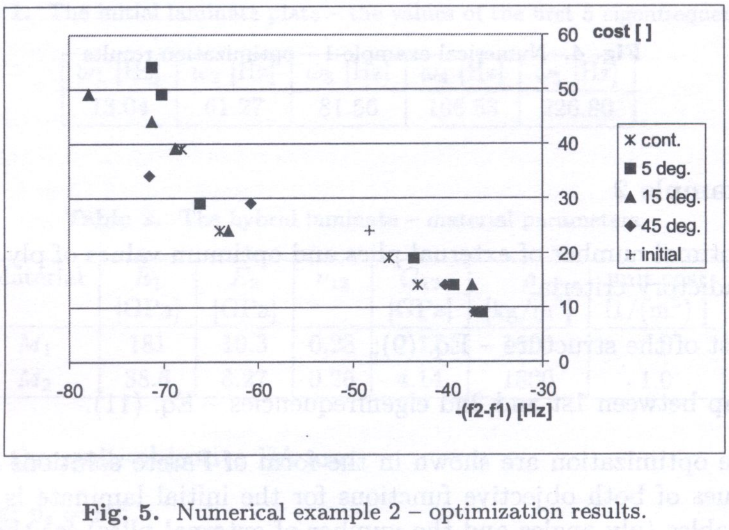

Fig. 5. Numerical example 2 – optimization results.

Table 4. Numerical example 2 – design variables values for optimal solutions

case	solution no.	stacking sequence	no of ext. plies	$f_2 - f_1$ [Hz]	cost [ ]
initial		(90/15/−45/45/−15/−90/−45/90/45)s	3	48.23	24
cont.	1	(−21.2/52.4/−55.5/−24.0/20.3/−65.9/52.8/76.7/−25.8)s	6	68.1533	39
	2	(−26.4/35.4/−53.4/−89.7/−52.6/−62.3/28.4/89.2/−81.3)s	3	64.0138	24
	3	(48.1/−11.9/−31.3/78.8/61.7/15.6/−10.2/39.4/3.5)s	2	46.163	19
	4	(48.1/−11.9/−4.8/−69.1/20.5/−62.3/−80.3/50.9/−72.2)	1	43.2417	14
	5	(28.2/−39.9/−44.3/42.3/37.2/36.3/−89.9/72.6/83.3)s	0	37.1053	9
5°	1	(−40/45/−10/55/−5/40/70/−85/−50)s	9	71.2261	54
	2	(−40/45/−10/55/−5/45/75/5/5)s	8	70.2639	49
	3	(−50/10/30/55/10/40/−50/−10/−20)s	4	66.1395	29
	4	(5/−50/−70/60/80/−45/−35/−75/10)s	2	43.5972	19
	5	(−35/−10/−45/−15/30/80/−70/−45/60)s	1	39.3754	14
	6	(30/−50/−30/−80/−45/50/−5/60/−60)s	0	36.4178	9
15°	1	(−45/30/15/−45/−45/75/90/90/75)s	8	77.8624	49
	2	(−60/−45/30/75/0/−15/60/−45/75)s	7	71.2431	44
	3	(45/30/−30/−75/45/−45/30/−60/90)s	6	68.757	39
	4	(30/−45/0/−75/15/−15/45/−30/45)s	3	63.2243	24
	5	(−60/−15/−30/90/−30/−45/0/90/45)s	1	37.4437	14
	6	(30/−45/−60/30/60/90/75/75/15)s	0	36.4361	9
45°	1	(−45/45/0/0/−45/45/90/90/90)s	5	71.5362	34
	2	(−45/0/0/−45/90/0/−45/90/45)s	4	60.78	29
	3	(−45/0/0/90/90/90/90/45/−45)s	1	40.3537	14
	4	(−45/0/0/−45/45/90/90/0/−45)s	0	37.172	9

6. Final conclusions

Hybrid laminates are not as popular as the simple laminates, but they ensure high efficiency with lower total cost of the laminate. The stacking sequence optimization gives the possibility to obtain the required properties of the laminate for given criteria. To avoid problems with calculation of the fitness function gradient the evolutionary algorithm has been employed.

To satisfy different and contradictory criteria the multi-objective attitude has been used. The coupling of multi-objective evolutionary algorithm with finite element method seems to be an effective and efficient method of the multi-optimization of laminates.

Due to the fact, that the number of laminas was discrete, the cost of the hybrid laminate was also a discrete variable. The plies orientation angles were considered as continuous as well as discrete variables. Positive optimization results were obtained for all presented tasks.

For the real problems the computation of the fitness function by means of the finite element method is the most time-consuming part of calculations. It can be significantly reduced by means of the distributed EA, which was presented in previous papers, e.g. [7, 9].

Acknowledgments

The research is partially financed from the Polish science budget resources as the research project and the Foundation for Polish Science (2005-2008).

REFERENCES

[1] S. Adali, A. Richte, V.E. Verijenko, E.B. Summers. Optimal design of symmetric hybrid laminates with discrete ply angles for maximum buckling load and minimum cost. *Composite Structures*, **32**: 409–415, 1995.
[2] S. Adali, V.E. Verijenko. Optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations. *Composite Structures*, **54**: 131–138, 2001.
[3] J.F. Aguilar Madeiraa, H. Rodriguesa, H. Pinaa. Multi-objective optimization of structures topology by genetic algorithms. *Advances in Engineering Software*, **36**: 21–28, 2005.
[4] J. Arabas. *Lectures on Evolutionary Algorithms* (in Polish). Wydawnictwa Naukowo-Techniczne, 2001.
[5] W. Beluch. Evolutionary identification and optimization of composite structures. In: *III European Conference on Computational Mechanics, ECCM 2006, Lisbon, CD-ROM*, 2006.
[6] W. Beluch, T. Burczyński. Evolutionary optimization of hybrid laminates. *Recent Developments in Artificial Intelligence Methods, AI-METH series*, 21–24, 2005.
[7] W. Beluch, T. Burczyński, W. Kuś. Distributed evolutionary algorithms in identification of material constants in composites. In: *Proceedings of the VIII Polish Conference Evolutionary Algorithms and Global Optimization, KAEiOG 2004, Kazimierz*, 1–8, 2004.
[8] T. Burczyński. *The Boundary Element Method in Mechanics* (in Polish). Wydawnictwa Naukowo-Techniczne, 1995.
[9] T. Burczyński, W. Kuś. Distributed and parallel evolutionary algorithms in optimization of nonlinear structures. In: *Proceedings of the 15th International Conference on Computer Methods in Mechanics CMM-2003*, 2003.
[10] C.M. Fonseca, P.J. Fleming. An overview of evolutionary algorithms in multiobjective optimization. *Evolutionary Computation*, **3**(1): 1–16, 1995.
[11] J. German. *The Basics of the Fibre-Reinforcement Composites' Mechanics* (in Polish). Cracow University of Technology Pub., 2001.
[12] T. Marler. *A Study of Multi-Objective Optimization Methods for Engineering Applications*. Ph.D. thesis, University of Iowa.
[13] *MSC/Nastran User's Guide*. MSC Software, 2000.
[14] A. Pegoretti, E. Fabbri, C. Migliaresi, F. Pilati. Intraply and interply hybrid composites based on e-glass and poly(vinyl alcohol) woven fabrics: tensile and impact properties. *Polymer International*, **53**: 1290–1297, 2004.
[15] R. Spallino, S. Rizzo. Multi-objective discrete optimization of laminated structures. *Mechanics Research Communications*, **29**: 17–25, 2002.
[16] S.W. Tsai, H.T. Hahn. *Introduction to Composite Materials*. Wesport Technomic Pub. Co., 1980.
[17] T. Uhl. *Computer-aided Identification of Constructional Models* (in Polish). Wydawnictwa Naukowo-Techniczne, 1997.