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Determination of the shear speed of soil triaxial testing based on fuzzy logic

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To design foundations, embankments and other soil structures, geotechnical engineers require methods of assessing engineering properties of soils. Some of the more complex phenomena that occur in soils have often been difficult to recreate in a laboratory: seismic activity, vibration, unsaturated condition, control of principal stresses etc. are areas which have proven difficult to replicate, despite their importance of being understood. This was partly due to the lack of test systems capable of reproducing these effects and the complexity of test systems that were developed to carry out such work. A number of advanced computer/software controlled systems allow the geotechnical engineer to perform the most complex test regimes via a user-friendly software interface. However, it is difficult to determine firstly parameters needed, e.g. shear speed in soil triaxial testing. In this paper we represent a new approach to determine this shear speed by solving the inverse problem using testing results obtained by the forward procedure. Direct search method, i.e. Adaptive Neuro-Fuzzy Inference System (ANFIS), is developed and applied to soil triaxial shear tests. It allows us to use the advanced sensor and actuator technologies in order to change the traditional triaxial shear apparatus from a mechanical system to a mechatronics system in next work.

Keywords: soil triaxial testing, shear speed, fuzzy logic, neuro-fuzzy inference system

1. SOIL TRIAXIAL TESTING

The principle of the triaxial test is illustrated in Fig. 1. The WF10056 TRITECH (presented in [6]) has been designed for the testing of triaxial soil specimens.

The sample, of cylindrical shape, is enclosed in a watertight cover and placed in a chamber that can be filled with fluid under a pressure p_c. An additional axial pressure q per unit of area can be applied to the top of the sample through a rigid head. Water may enter or leave the sample through a porous stone in the bottom, provided the valve V is left open. The pressure of the water in the sample may be measured by means of a pressure gage connected to the discharge pipe above the valve V. An extensometer is provided to measure the strain of the sample in the vertical direction. The sample is subjected to the supplementary axial pressure q per unit of area. The axial pressure is increased until the specimen fails. During the application of this pressure the valve V may be either

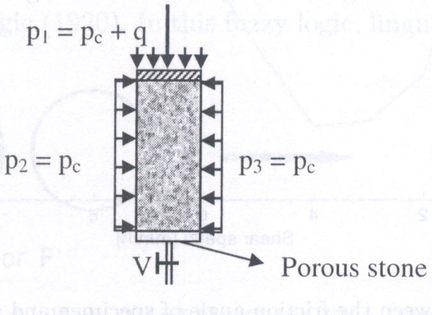

Fig. 1. Diagram illustrating principle features of triaxial-test apparatus

closed or open. The results of a test with the valve closed represent the equivalent of a consolidated-quick-shear test. If the axial pressure is applied very slowly, so that the water within the sample has time to adjust itself to the change in stress, the test corresponds to a slow-shear test made with box-shear apparatus. The values obtained in this manner are referred to as slow-test values. The sample fails along inclined shear planes. The stress conditions on the surface of failure can be determined by computation or by an equivalent graphical construction known as Mohr's diagram.

With the introduction of closed loop servo-controlled pneumatic systems as proposed by Wykeham Farrance Company [5], the stress path of soil can be accurately reproduced and results collected and processed in a format that is easy to interpret. The use of the stress path test in a laboratory enables field changes — past, present and future — to be modeled. The laboratory Stress Path Test allows an engineer to replicate the changes in stress conditions experienced during excavation, construction works or those that occur due to natural events. The Stress Path Triaxial test can be upgraded and improved by the accurate measurement of sample deflections and stiffness evaluation of the soil using on sample strain transducers and Bender elements. The actuator has an integral displacement transducer, which allows tests to be run under load and displacement control. To produce the correct test conditions for an unsaturated soil sample, for example, we must be able to: control the pore air pressure within the sample (independently of the pore water pressure); deal with the negative pore water pressure (or suction) within the sample during test; successfully measure the volume change of the sample. The software, in Tritech Machine, allows us, in this case, to select the shear rate speed (within the range 0.00001 to 9.99999 mm per minute) we wish to run — 'soil kinematic testing'. Thus, advanced controlled systems allow a geotechnical engineer to perform the most complex test regimes via a user-friendly software interface. Remaining task is how we can determine the shear speed which is really dependent on the load conditions and type of sample expressed by different soil parameters. It is difficult/impossible to make well-defined mathematical model for this nonlinear relationship of them and the inverse problem needs to be solved.

Soils in general are non-elastic materials characterized by high-nonlinear relationship of many factors. These include the following: magnitude and direction of the imposed stress changes; the way in which the stress on the sample changes; previous history of loading, whether by natural causes or changes imposed by man. For example, the high-nonlinear relationship between the friction angle of specimen and shear speed of apparatus via student testing results obtained from a modernized triaxial shear apparatus (in laboratory of the Department of Geotechnics, Faculty of Technical Sciences, UWM) is shown in Fig. 2.

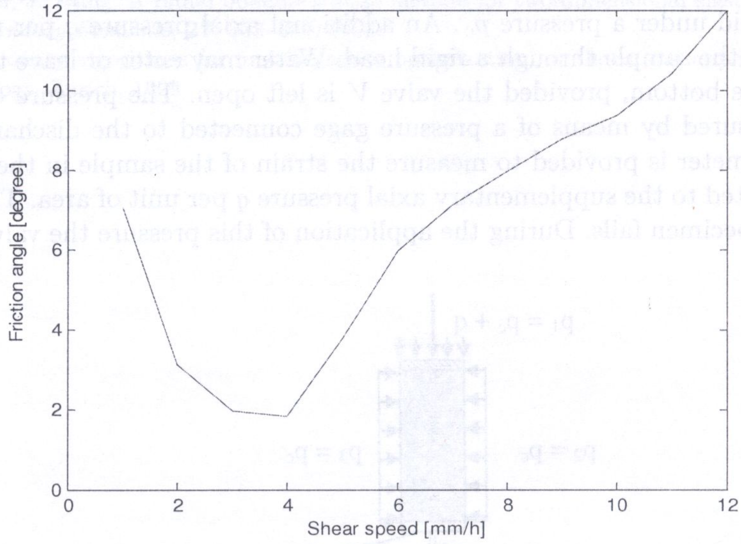

Fig. 2. Relationship between the friction angle of specimen and shear speed of apparatus

2. Inverse problems

Generally, inverse theory is concerned with the problem of making inferences about physical systems from data in order to determine causes for a desired or an observed effect. It is, for example, the identification of physical parameters from observations of the evolution of the system. In a computational approach to an inverse problem, the numerical solution to all elliptic boundary value problems (such as the Poisson and Laplace problems), for example, can be written in the form of a set of linear equations,

$$\mathbf{A}\mathbf{z} = \mathbf{u}, \qquad \mathbf{z} \in \mathbf{Z}, \ \ \mathbf{u} \in \mathbf{U}, \tag{1}$$

where $\mathbf{z}$ is solution vector, $\mathbf{u}$ is the vector of input data, and $\mathbf{A}$ is the transfer matrix between $\mathbf{z}$ and $\mathbf{u}$. The forward problem is then simply posed as solving equation (1) for $\mathbf{u}$ given $\mathbf{z}$. Likewise, the inverse problem is to determine $\mathbf{z}$ given $\mathbf{u}$. Usually, we first assume the values of parameters in an appropriate manner, next, carry out analysis of the forward problem. Results obtained are compared with the measured data given as additional information, and then parameter values are modified so that an appropriate cost function is minimized in an iterative manner. The simplest of Tikhonov's methods presented by Heinz W. Engl [2], for example, consider minimizing the following functional for the solution vector $\mathbf{z}$,

$$M\{\mathbf{A}, \mathbf{z}, \mathbf{u}\} = \|\mathbf{A}\mathbf{z} - \mathbf{u}\|^2 + \alpha \|\mathbf{I}\mathbf{z}\|^2, \qquad \alpha > 0, \tag{2}$$

which yields:

$$\mathbf{z} = \left[\mathbf{A}^T\mathbf{A} + \alpha\mathbf{I}\right]^{-1}\left[\mathbf{A}^T\mathbf{u}\right]. \tag{3}$$

This is the Tikhonov method with zero-order regularization with regularization parameter α. The zero- order corresponds to the inclusion of the identity matrix $\mathbf{I}$ in Eq. (3). It is, from the computing point of view, similar to the inversion of a neural net to try to find an input pattern that generates a specific output pattern with the existing connection. To find this input, the deviation of each output from the desired output is computed as error. This error value is used to approach the target input in input space step by step. In this paper, a new approach based on Neuro-Fuzzy Inference System is represented as follows.

3. Fuzzy logic

Beginning with the Chinese philosopher Trang Chau (369–298 BC), who developed an alternative to the binary idea of Confucianism, against the "the being or the non-being", "true or false", (**T** or **F**) confirmation. In his philosophy, true contains an amount of false and false contains an amount of true; they have common sense which is subjective and approximate in nature. Thus, true and false are united and undivided in the term of truth, that is to say — a single-nary philosophy (see Tran [4]) which is represented graphically in Fig. 3.

The many-valued logic based on the single-nary philosophy, which Zadeh (1965) [7] takes as a basis model for his model of linguistic reasoning with vague statements named **fuzzy logic**, is Łukasiewicz's infinitely valued logic (1920). In this fuzzy logic, linguistic variable is viewed as a label

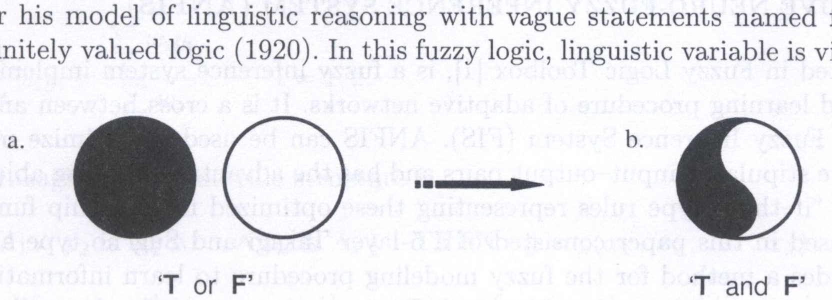

Fig. 3. From 'T or F' to 'T and F' philosophy

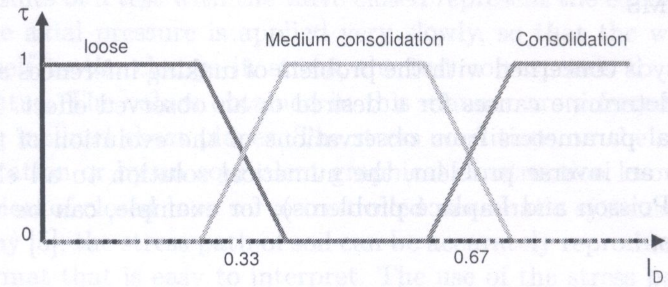

Fig. 4. Idea of "Truth functions/Membership functions"

of a granule, which is a fuzzy set of points having the form of a clump of elements drawn together by similarity. The way around this is to make the states "fuzzy", that is, allow them to change gradually from one state to the next. We can define the input consolidation states (I_D), for example, using "truth (τ)/membership functions (μ)", see Tran [3], such as the one presented in Fig. 4.

Basic operations on fuzzy sets are represented as follows: the *standard complement* of a fuzzy set A in X, $\neg A$ (corresponding to the negation "NOT"), is defined by

$$\tau_{\neg A}(x) = 1 - \tau_A(x), \quad \forall x \in X. \tag{4}$$

Fuzzy intersections

The standard intersection of two fuzzy sets A, B in X, $A \cap B$ (corresponding to the connective "AND"), is defined by

$$\tau_{A\cap B}(x) = \tau_A(x) \wedge \tau_B(x), \quad \forall x \in X. \tag{5}$$

Fuzzy unions

The standard union of two fuzzy sets, A, B in X, $A \cup B$ (corresponding to the connective "OR") is defined by

$$\tau_{A\cup B}(x) = \tau_A(x) \vee \tau_B(x), \quad \forall x \in X. \tag{6}$$

Fuzzy implication

The standard implication of two fuzzy sets, A, B in X, is defined by

$$\tau_{A\to B}(x) = \min\left(1, 1 - \tau_A(x) + \tau_B(x)\right). \tag{7}$$

Here, symbols $\wedge$ and $\vee$ denote minimum operator and maximum operator, respectively. These definitions can be extended to any finite number of fuzzy sets.

4. An adaptive neuro-fuzzy inference system (ANFIS)

ANFIS, presented in Fuzzy Logic Toolbox [1], is a fuzzy inference system implemented within the architecture and learning procedure of adaptive networks. It is a cross between an artificial Neural Network and a Fuzzy Inference System (FIS). ANFIS can be used to optimize membership functions to generate stipulated input–output pairs and has the advantage of being able to subsequently construct fuzzy "if–then" type rules representing these optimized membership functions. The ANFIS structure used in this paper consisted of a 5-layer Takagi and Sugeno type architecture. This technique provides a method for the fuzzy modeling procedure to learn information about a data set, in order to compute the membership/truth function parameters that best allow the associated fuzzy inference system to tract the given input-output data. A typical example is shown in Fig. 5.

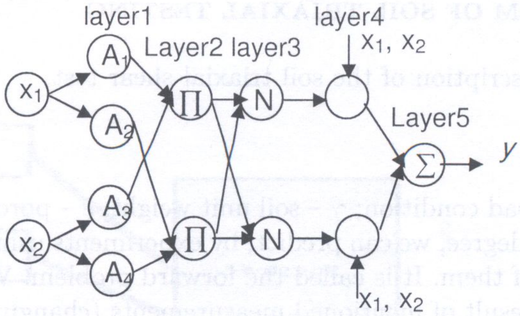

Fig. 5. A 5-layer ANFIS structure

Rather than choosing the parameters associated with a given membership/truth function arbitrarily, these parameters could be chosen so as to tailor the membership/truth functions to the input-output data in order to account for these types of variations in the data values. Here, two inputs (x_1, x_2), for example, are used and one output (y) i.e. that there is only a single output.

In the first layer, all nodes are adaptive, μ_{Ai} is the degree of the membership of the input to the fuzzy membership/truth function represented by the node,

$$O_{1i} = \mu_{Ai}(x)\,, \qquad i = 1,2,3,4, \tag{8}$$

where $O_{li}$is the output of the node i in a layer l.

In the second layer the nodes are fixed (i.e. not adaptive). Nodes in this layer are labeled Π and multiply the signal before outputting. The outputs are given by

$$O_{2i} = w_i = \mu_{Ai}(x_1)\,\mu_{Aj}(x_2), \qquad i = 1,2; \quad j = i + 2. \tag{9}$$

Each node output in this layer represents the firing strength of the rule.

In the third layer, every node is also fixed and labeled with an N and performs a normalization of the firing strength from the previous layer. The output of each node is given by

$$O_{3i} = \overline{w}_i = \frac{w_i}{w_1 + w_2}\,, \qquad i = 1,2. \tag{10}$$

In the fourth layer, all nodes are adaptive. The output of a node is the product of the normalized firing strength and a first order polynomial and is given by

$$O_{4i} = \overline{w}_i y_i = \overline{w}_i(p_i x_1 + q_i x_2 + r_i), \qquad i = 1,2, \tag{11}$$

where $\{p_i x_1\,, q_i x_2\,, r_i\}$ (p, q, r are all constants) is the modifiable parameter set, referred to as *consequent parameters* since they deal with the *then* part of the fuzzy rule.

Finally, layer 5 is a single node labeled with Σ which indicates that the function is that of computing the overall output as the summation of all incoming signals,

$$O_{5i} = y = \sum_i \overline{w}_i y_i = \frac{\sum_i w_i y_i}{\sum_i w_i}\,, \qquad i = 1,2. \tag{12}$$

The general Takagi and Sugeno rule structure is

$$\text{IF} \quad f\,(\,x_1 \text{ is } A_1 \wedge x_2 \text{ is } A_2 \wedge \ldots \wedge x_k \text{ is } A_k\,) \quad \text{THEN} \quad y = g(x_1\,, x_2\,, \ldots\,, x_k)\,. \tag{13}$$

Here f is a logical function that connects the sentences forming the condition, y is the output, and g is a function of the inputs.

5. The inverse problem of soil triaxial testing

Usually, given a complete description of the soil triaxial shear test,

$$\{v-\gamma\,,\, e\,,\, \varepsilon\,,\, I_L\,,\, w\,,\, q\}\,, \tag{14}$$

where, v – shear speed; q – load condition; γ – soil unit weight; e – porosity index; ε – straining; w – humidity and I_L – plasticity degree, we can predict, by experiments using geo-mechanical theory, the outcome of any correlation of them. It is called the forward problem. Whereas, the inverse problem consists of using the actual result of mentioned measurements (changing in admissible spaces Γ, E, Σ, Ξ, W respectively) to infer the value of shear speed, v;

$$\{\gamma\in\Gamma\,,\, e\in E\,,\, \varepsilon\in\Sigma\,,\, I_L\in\Xi\,,\, w\in W\} \to v. \tag{15}$$

Here, we have to answer the following questions "how is the v with these soil parameters?" and "what soil parameters reply to this v?". These two questions are called the **forward** and **inverse procedure**, respectively. Forward procedure consists of: (1) setting shear speed depending on load condition and soil parameters (it should be performed by Fuzzy Inference techniques using fuzzy logic which is beyond the limit of this paper), (2) fulfilling a testing plan in order to produce the information on the physical correlations between shear values and a set of soil parameters. A set of training samples, presented in Table 1, is collected from measurements for Mazury-clay with summary of testing data, from standard tests.

Table 1. Testing data set

γ [kN/m^3]	e [–]	ε [%]	w [%]	I_L [%]
18.75	0.914	2.47	32.99	0.04
18.56	0.88	3.42	29.90	−0.03
18.23	0.98	3.00	34.02	0.02
18.04	0.963	8.47	31.33	0.00
18.06	0.978	3.87	32.69	0.01
18.06	0.949	7.93	31.03	−0.03
18.08	0.968	4.36	32.50	−0.03
18.08	0.965	1.90	32.03	−0.02
18.17	0.929	9.53	30.27	−0.04
18.63	0.917	8.58	32.35	0.02
18.63	0.918	8.82	32.44	0.02
18.17	0.988	9.02	33.95	0.06

Inverse procedure consists of: processing the measured results using ANFIS to solve the inverse problem. The model of Fuzzy Inference System (FIS) structure used in this paper is shown in Fig. 6.

By using the measurement data, an ANFIS is trained with sufficient number of data points. The network-type shown in Fig. 7 is used to interpret the input-output map.

Three non-linear truth/membership functions for input1, for example, are shown in Fig. 8.

In the training phase, the membership/truth functions and the weights will be adjusted so that the required minimum error is satisfied. Consequently, the trained ANFIS can be utilized in order to provide acceptable inverse solutions of the soil triaxial testing. In the simulation, the difference between desired values of shear speed and test values resulted from ANFIS are shown in Fig. 9:

The successful mapping $\{\gamma\in\Gamma\,,\, e\in E\,,\, \varepsilon\in\Sigma\,,\, \mathrm{I}_L\in\Xi\,,\, w\in W\} \to v$ including the advanced sensor and actuator technologies enables us to change, using software interface, the traditional soil triaxial shear apparatus from a mechanical system to a mechatronics system in the future.

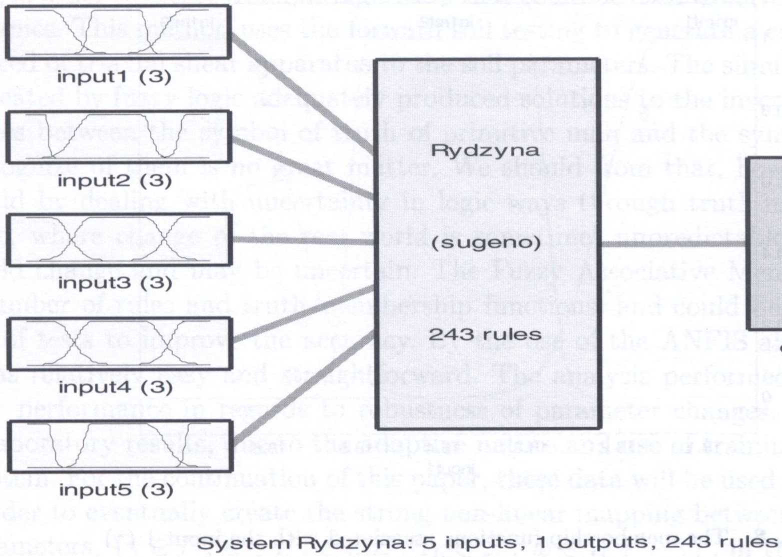

Fig. 6. The based model of FIS structure

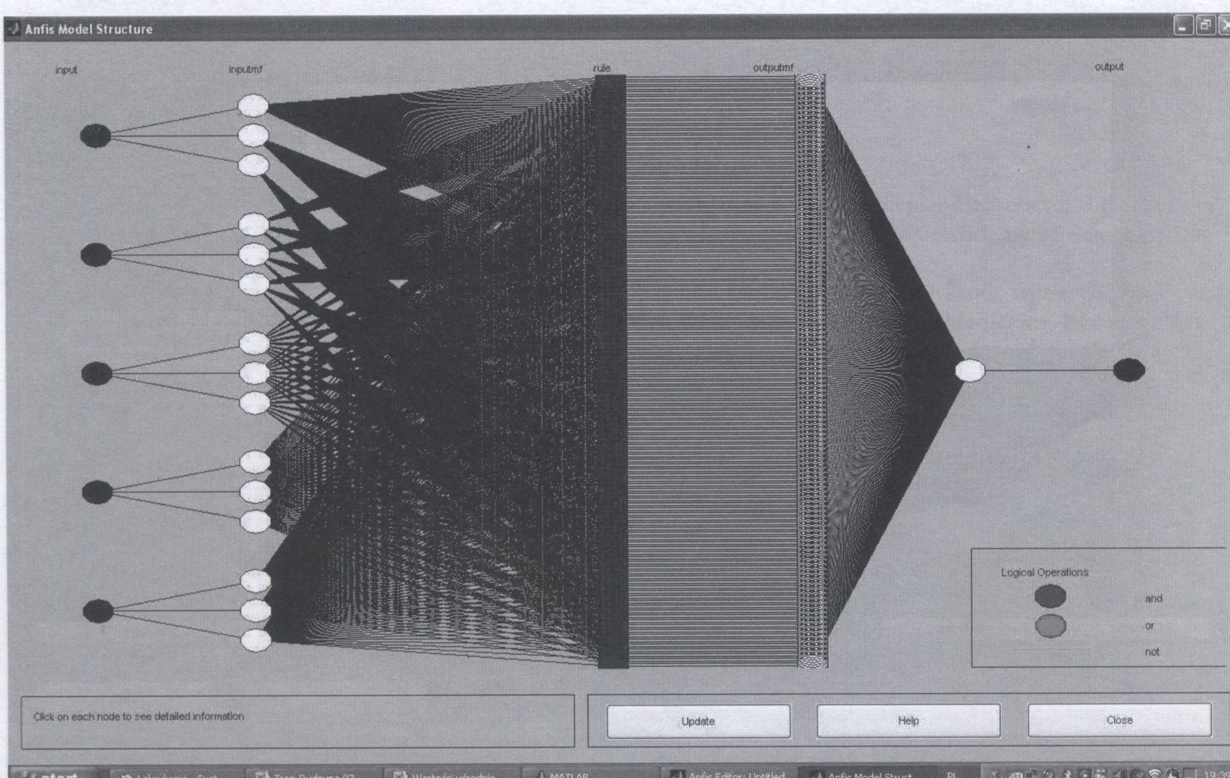

Fig. 7. The Fuzzy Associative Memory (FAM) structure of the Fuzzy Inference System

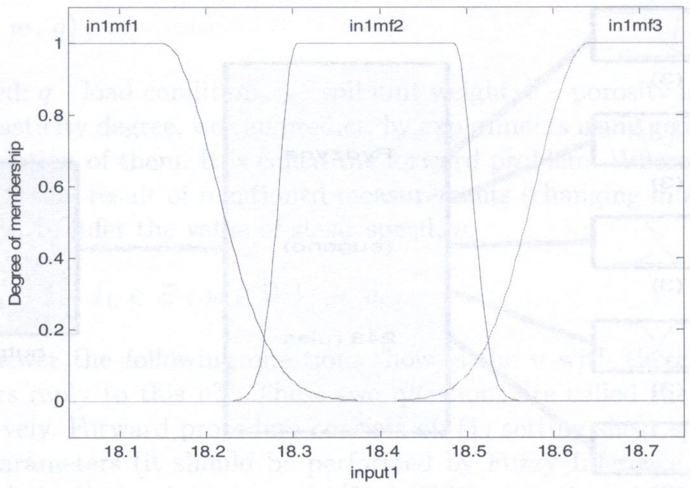

Fig. 8. The membership functions associated with the input 1 (γ)

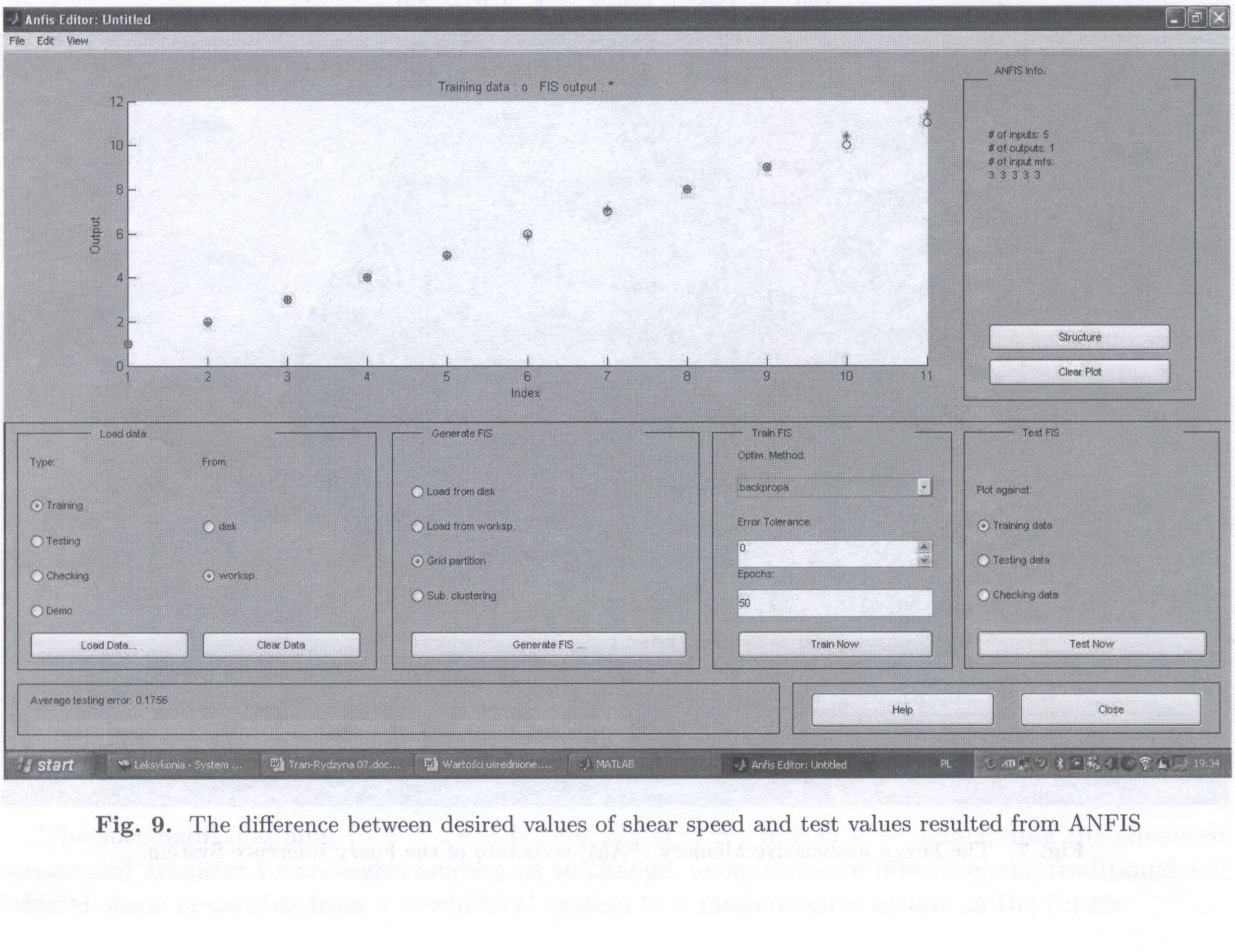

Fig. 9. The difference between desired values of shear speed and test values resulted from ANFIS

6. Conclusions

In this paper, ANFIS algorithm has been utilized to establish the inverse mapping for soil triaxial shear testing in order to create a simple algorithm that could be used to solve the non-linear problems of soil mechanics. This method uses the forward soil testing to generate a collection of data relating the shear speed of triaxial shear apparatus to the soil parameters. The simulation indicated that the mappings created by fuzzy logic adequately produced solutions to the inverse problems. Here, there are differences between the symbol of truth of primitive man and the symbol of truth of civilized man. To recognize of them is no great matter. We should from that, however, better understand the real world by dealing with uncertainty in logic ways through truth-measures and fuzzy rules – fuzzy logic, where change of the real world is sometimes unpredictable, and our beliefs about the real world change and may be uncertain. The Fuzzy Associative Memory (FAM) utilized the minimum number of rules and truth/membership functions, and could be increased by increasing the number of tests to improve the accuracy. By the use of the ANFIS algorithm, the training of the FAM was relatively easy and straightforward. The analysis performed showed that the FAM had superior performance in regards to robustness of parameter changes. ANFIS was chosen, for analysis of laboratory results, due to the adaptive nature and use of training data to create a fuzzy inference system. For the continuation of this paper, these data will be used to generate and train an ANFIS in order to eventually create the strong non-linear mapping between the soil shear strength and soil parameters, $\{\gamma \in \Gamma\,, e \in E\,, \varepsilon \in \Sigma\,, \mathrm{I}_L \in \Xi\,, w \in W\} \rightarrow \tau_f$, in the next time. Note that, an advantage of the neural network approach is the case, which gives the possibility of retraining and performance improving, as additional data are acquired. In view of the limited number of case records from Triaxial tests considered, it would be desirable to reassess the neural network model, as further case records become available. These data can be readily included in the neural-network training and testing data, to improve the modeling of this testing further. However, in order to prove the effectiveness of this technique for real-life situations; it will be necessary to prove the concept from experimental studies using a more detailed model.

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