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New numerical method for mechanics of continuous media[1]

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A new, original numerical method of free particles, for mechanics of continuous media, has been developed. The free particle hydrocodes (HEFP), based on this method, are the powerful tools that can be used to simulate, in the sense of computational physics, events with very high dynamic effects. In the paper, computational simulation of several attractive and practically important problems like: processes of the detonation, high velocity impacts and hypervelocity planetary impacts, shaped charge jet formation, explosive forming of projectiles, penetrating of the armour plate, is posed. All of them include shocks, very large deformations of solids, processes of cratering with impact jets generation and targets penetration. The method of free particles is a very useful for magnetohydrodynamical (MHD) simulation, too. It is possible to simulate ideal MHD and non-ideal MHD processes, and such exemplary results are also presented. In the paper, physical, mathematical and numerical models as well as results of some complex, unsteady, spatially two-dimensional simulations are presented.

1. INTRODUCTION

Initial and boundary value problems of highly dynamic events like collisions and penetrations, large dynamical deformations of solids, processes of detonation of explosives and MHD phenomena belong to the most complicated problems of the mechanics of continuous media. It results from the following reasons:

- nonstationarity and multidimensional character of the problems,
- quantity and complexity of physical processes concomitant with the collisions and deformation of solids, detonation of explosives, and MHD plasma evolution,
- complicated mathematical description of these processes, consistent with the physical reality,
- complicated and continually extended description of properties of highly deformed materials.

Several exemplary solutions of especially attractive and important problems will be presented [1–5, 14–16, 19], namely:

- formation of the cumulative jet,
- explosive forming of projectile,
- penetration of a target by the deformable body,

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- planetary impact,
- MHD instability.

The physical models of these phenomena include:

- general gas dynamic equations and semi-empirical equations of state for the description of spatial process of detonation of explosives,
- equations of the elastic/visco-plastic or elastic/plastic theory together with Steinberg model of semi-empirical material characteristic, and phenomenological model of the crack formation for the description of the deformation process of the solid liners and projectiles,
- general equations of hydrodynamics with gravity,
- equations of ideal and non-ideal magnetohydrodynamics (MHD).

The appropriate for each case set of transient, nonlinear partial differential equations in two dimensions has been solved using an original numerical method which enables:

- use of special 9-points numerical schemes,
- calculating problems with large deformations, including the possibility of fragmentation of the investigated objects,
- treating of the boundary conditions on the moving and curvilinear surfaces,
- linking of the solutions on the interfaces of different media,
- obtaining of the stable solutions for long times etc.

2. Physical models of the dynamical processes

From the previous experience (see [1–4]) we can state that, in order to achieve good agreement between the numerical simulation and experiment, the physical description of the processes of shaped charge jet formation, explosive forming of projectiles and penetrating of the armour plate should include:

- the set of general equations of the elastic/visco-plastic theory with semi-empirical material formulas,
- hydrodynamic theory of detonation with the JWL type equation of state.

The set of conservation equations in two dimensions can be specified as follows:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right) = 0\,, \tag{1}$$

$$\rho\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\partial p}{\partial r} + \frac{\partial S_{rr}}{\partial r} + \frac{\partial S_{rz}}{\partial z} + \frac{S_{rr} - S_{\phi\phi}}{r}\,, \tag{2}$$

$$\rho\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\partial p}{\partial z} + \frac{\partial S_{rz}}{\partial r} + \frac{\partial S_{zz}}{\partial z} + \frac{S_{rz}}{r}\,, \tag{3}$$

$$\rho\frac{\mathrm{d}E}{\mathrm{d}t} = -p\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right) + S_{rr}\frac{\partial u}{\partial r} + S_{\phi\phi}\frac{u}{r} + S_{zz}\frac{\partial v}{\partial z} + S_{rz}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right). \tag{4}$$

Constitutive relations of the elastic/visco-plastic material are taken in the form:

$$\frac{\partial S_{rr}}{\partial t} = 2\mu\left[\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right)\right] + S_{rz}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right) - \frac{\mu}{\eta}\Phi S_{rr}\,, \tag{5}$$

$$\frac{\partial S_{\phi\phi}}{\partial t} = 2\mu\left[\frac{u}{r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right)\right] - \frac{\mu}{\eta}\Phi S_{\phi\phi}\,, \tag{6}$$

$$\frac{\partial S_{zz}}{\partial t} = 2\mu\left[\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right)\right] - S_{rz}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right) - \frac{\mu}{\eta}\Phi S_{zz}\,, \tag{7}$$

$$\frac{\partial S_{rz}}{\partial t} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) - \frac{1}{2}(S_{rr} - S_{zz})\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right) - \frac{\mu}{\eta}\Phi S_{rz}\,, \tag{8}$$

where:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + v\frac{\partial}{\partial z}\,, \tag{9}$$

$$\Phi = 1 - \sqrt{\frac{2}{3S_{ik}S_{ik}}}\,Y\,, \qquad \Phi \geq 0\,, \tag{10}$$

$$S_{ik}S_{ik} = S_{rr}^2 + S_{\phi\phi}^2 + S_{zz}^2 + 2S_{rz}^2\,. \tag{11}$$

Equation of state for liner and casing is proposed as follows,

$$p = k_1 x + k_2 x^2 + k_3 x^3 + \gamma_0 \rho_0 E\,, \tag{12}$$

where

$$x = 1 - \frac{\rho_0}{\rho_S}\,, \qquad k_2 = 0 \quad \text{if } x < 0\,. \tag{13}$$

The dependence of dynamic yield strength Y and shear modulus on plastic strain, pressure and temperature has been adopted according to the Steinberg model [7,8]:

$$Y = Y_0(1 + \beta\varepsilon^p)^n\left[1 + bp\left(\frac{\rho_0}{\rho_S}\right)^{\frac{1}{3}} - h(T - 300)\right], \tag{14}$$

$$Y_0(1 + \beta\varepsilon^p)^n \leq Y_{\max}\,, \qquad Y = 0 \quad \text{if } T > T_m\,, \tag{15}$$

$$\mu = \mu_0\left[1 + bp\left(\frac{\rho_0}{\rho_S}\right)^{\frac{1}{3}} - h(T - 300)\right], \tag{16}$$

$$T_m = T_{m0}\left(\frac{\rho_0}{\rho_S}\right)^{\frac{2}{3}}\exp\left[2\gamma_0\left(1 - \frac{\rho_0}{\rho_S}\right)\right], \tag{17}$$

$$\varepsilon^p = 0.4714\left[\left(\varepsilon_{rr}^p - \varepsilon_{zz}^p\right)^2 + \left(\varepsilon_{rr}^p - \varepsilon_{\phi\phi}^p\right)^2 + \left(\varepsilon_{zz}^p - \varepsilon_{\phi\phi}^p\right)^2 + \frac{3}{2}\left(\varepsilon_{rz}^p\right)^2\right]^{\frac{1}{2}}, \tag{18}$$

$$\frac{\mathrm{d}\varepsilon_{ik}^p}{\mathrm{d}t} = \frac{\Phi}{2\eta}S_{ik}\,, \tag{19}$$

$$\eta = \eta_0\exp\left(\frac{T_0}{T}\right). \tag{20}$$

The model of forming and growth of cracks has been taken in the same form as in [9]:

$$\frac{\mathrm{d}V_C}{\mathrm{d}t} = -k\,\mathrm{sign}(p)\left(|p| \quad \sigma_0\frac{V_{C1}}{V_C + V_{C1}}\right)(V_C + V_{C0}) \qquad \text{for } |p| > \sigma_0\frac{V_{C1}}{V_C + V_{C1}}\,, \tag{21}$$

$$\frac{\mathrm{d}V_C}{\mathrm{d}t} = 0 \qquad \text{for } |p| < \sigma_0\frac{V_{C1}}{V_C + V_{C1}}\,. \tag{22}$$

Here, k, V_{C0}, V_{C1} and σ_0 are constant parameters to be determined experimentally.

The density of the solid phase can be obtained from the relation

$$V_C = \frac{1}{\rho} - \frac{1}{\rho_S} \tag{23}$$

where V_C is the specific volume of cracks and ρ_S is the density of the solid phase in a medium with cracks.

It is very important to take into account the proper dependence of viscosity on temperature $\eta = \eta(T)$ and the influence of the crack volume growth on Y, μ and η. The best comparison between the numerical simulation and experiment [1,3] has been obtained for the relation [10]:

$$\eta = 5 \times 10^{-3} \exp\left(\frac{4380}{T}\right) \, [\mathrm{Pa \cdot s}] \qquad \text{(for copper).} \tag{24}$$

T is temperature in the Kelvin scale.

The relations for Y, μ and η in dependence of the crack volume have been adopted from [9,11] in the following form,

$$Y^T = YF\,, \qquad \mu^T = \mu F\,, \qquad \eta^T = \eta F\,, \tag{25}$$

where

$$F = \frac{V_{C1}}{V_C + V_{C1}} \qquad \text{– for aluminum and steel,} \tag{26}$$

$$F = \exp(-7\rho V_C) \qquad \text{– for copper.} \tag{27}$$

The set of the hydrodynamic equations describing the detonation products is taken in the form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right) = 0\,, \tag{28}$$

$$\rho\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\partial p}{\partial r}\,, \tag{29}$$

$$\rho\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\partial p}{\partial z}\,, \tag{30}$$

$$\rho\frac{\mathrm{d}E}{\mathrm{d}t} = -p\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right). \tag{31}$$

The equation of state of the detonation products of condensed explosive (the JWL type) is taken in the form

$$p = A\left(1 - \frac{\delta}{R_1 V}\right)e^{-R_1 V} + B\left(1 - \frac{\delta}{R_2 V}\right)e^{-R_2 V} + \delta\rho E \tag{32}$$

where

$$V = \frac{\rho_e}{\rho}\,,$$

while p, ρ, E and ρ_e denote pressure, density, internal energy of the detonation products per unit mass and density of the explosive, respectively.

Another class of problems that we want to investigate is connected with planetary impacts [14–16, 19]. It is impossible to compare theoretical and experimental results directly, therefore we ought to test our simulation indirectly, for instance by comparing with seismological data like the travel time through the Earth of the P-waves (more correctly — PKIKP waves) [12].

When the local problems of the asteroids impact onto the Earth are studied (crater and jet formation, asteroid with diameter about 10 km), mathematical description of dynamical processes must make use of elastic-plastic material model of the response of the impacted Earth. In the cylindrical system of coordinates the following set of equations, with constant gravity, has been applied to formulate the problem mathematically:

– Conservation equations:

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right) = 0, \tag{33}$$

$$\rho\frac{du}{dt} = -\frac{GM_0\rho}{R_0^2}\frac{r}{R} - \frac{\partial p}{\partial r} + \frac{\partial S_{rr}}{\partial r} + \frac{\partial S_{rz}}{\partial z} + \frac{S_{rr} - S_{\phi\phi}}{r}, \tag{34}$$

$$\rho\frac{dv}{dt} = -\frac{GM_0\rho}{R_0^2}\frac{z}{R} - \frac{\partial p}{\partial z} + \frac{\partial S_{rz}}{\partial r} + \frac{\partial S_{zz}}{\partial z} + \frac{S_{rz}}{r}, \tag{35}$$

$$\rho\frac{dE}{dt} = -p\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right) + S_{rr}\frac{\partial u}{\partial r} + S_{\phi\phi}\frac{u}{r} + S_{zz}\frac{\partial v}{\partial z} + S_{rz}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right). \tag{36}$$

– Constitutive relations:

$$\frac{\partial S_{rr}}{\partial t} = 2\mu\left[\frac{\partial u}{\partial r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right)\right] + S_{rz}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right), \tag{37}$$

$$\frac{\partial S_{\phi\phi}}{\partial t} = 2\mu\left[\frac{u}{r} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right)\right], \tag{38}$$

$$\frac{\partial S_{zz}}{\partial t} = 2\mu\left[\frac{\partial v}{\partial z} - \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right)\right] - S_{rz}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right), \tag{39}$$

$$\frac{\partial S_{rz}}{\partial t} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) - \frac{1}{2}(S_{rr} - S_{zz})\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right). \tag{40}$$

The von Mises limits of elasticity are assumed,

$$S_{rr}^2 + S_{\phi\phi}^2 + S_{zz}^2 + 2S_{rz}^2 \leq \frac{2}{3}Y^2, \tag{41}$$

where:

$$Y = \min(C_0 + \alpha p, Y_{vm}). \tag{42}$$

Here, C_0 denotes cohesion, α is the slope of the Mohr-Coulomb surface and Y_{vm} is the von Mises limit for compression.

Equation of state has been assumed in the form proposed by Isenberg and Schuster [14],

$$p = K_m x - (K_m - K_I)\mu^*\left[1 - \exp\left(-\frac{x}{\mu^*}\right)\right] + \gamma\rho E, \tag{43}$$

where:

$$x = \frac{\rho - \rho_0}{\rho_0}, \tag{44}$$

while K_I, K_m are the initial and maximum bulk moduli, respectively, and μ^* is the empirical factor.

The dynamical effects of high velocity planetary collision with a large asteroid (radius > 55 km) [14, 15, 19] and sub-giant impact with a spherical asteroid (diameter about 1600 km), with regard to the global response of the Earth, have been computed on the basis of the hydrodynamic theory of impact taking into account gravitational force:

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right) = 0, \tag{45}$$

$$\rho\frac{du}{dt} = -\frac{GM_0\rho}{R_0^2}\frac{r}{R} - \frac{\partial p}{\partial r}, \tag{46}$$

$$\rho\frac{dv}{dt} = -\frac{GM_0\rho}{R_0^2}\frac{z}{R} - \frac{\partial p}{\partial z}, \tag{47}$$

$$\rho\frac{dE}{dt} = -p\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r}\right). \tag{48}$$

The strength of material is not taken into account. The global spherical symmetry of the undisturbed Earth (before impact) is assumed. Therefore the initial (undisturbed) density distribution

$$\rho_0 = \rho_0(R), \qquad 0 < R < R_0, \tag{49}$$

follows from the PREM model by Dziewonski and Anderson [13]; R denotes the radius of the undisturbed Earth. We use the Gruneisen parameter distribution,

$$\gamma = \gamma(R), \qquad 0 < R < R_0, \tag{50}$$

determined by Stacey [18]. The Gruneisen parameter is supposed to be undisturbable and it is always given by Stacey data.

The Earth material is assumed to obey equation of state (EOS) in the form similar to Murnaghan pressure-density relation with the Gruneisen term

$$p - p_0 = \frac{K_0}{K_0'}\left[\left(\frac{\rho}{\rho_0}\right)^{K_0'} - 1\right] + \gamma\rho(E - E_0). \tag{51}$$

Here, p is pressure, ρ is density, and E denotes specific energy per unit of mass (all are taken at a given point and at a specified time instant). The undisturbed bulk modulus K_0 depends linearly on the undisturbed pressure p_0,

$$K_0 \equiv K(p_0) = K_{00} + K_0' p_0 \tag{52}$$

with

$$K_{00} = 2.25 \times 10^{11}\,\mathrm{Pa} \qquad \text{and} \qquad K_0' = 3.35\,. \tag{53}$$

The numerical coefficients in this equation are chosen to be equal to the mean values over the whole Earth mantle [17, 20]. In the original Murnaghan equation, both K_0 and K_0' are taken at pressure equal to zero.

The undisturbed pressure p_0 is determined from the hydrostatic equation

$$\frac{\mathrm{d}p_0}{\mathrm{d}R} = -\frac{G\, m_0(R)\, \rho_0(R)}{R^2} \tag{54}$$

where $G = 6.672 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$ is the gravity constant and $m_0(R)$ is the partial mass within an undisturbed sphere

$$m_0 = 4\pi \int_0^R \rho_0(x) x^2\, \mathrm{d}x\,. \tag{55}$$

The density of impactor is assumed to equal $2.6\,\mathrm{gcm^{-3}}$; the other material parameters of impactor are assumed similar to that of the Earth mantle.

The equations describing the MHD process in the Z-pinch are taken as follows [5]:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\nabla\cdot\mathbf{w} = 0\,, \tag{56}$$

$$\rho\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} = \nabla\sigma + \frac{1}{c}\mathbf{j}\times\mathbf{B}\,, \tag{57}$$

$$\rho c_v \frac{\mathrm{d}T_i}{\mathrm{d}t} = \sigma^i\cdot\nabla\mathbf{w} + z\rho c_v\frac{T_e - T_i}{\tau_{ei}} + W_i\,, \tag{58}$$

$$z\rho c_v\frac{\mathrm{d}T_e}{\mathrm{d}t} = \sigma^e\cdot\nabla\mathbf{w} - \rho c_v T_e\frac{\mathrm{d}z}{\mathrm{d}t} - \rho\frac{\mathrm{d}Q_i}{\mathrm{d}t} + \eta j^2 - z\rho c_v\frac{T_e - T_i}{\tau_{ei}} + W_e\,, \tag{59}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{w}\times\mathbf{B}) - \frac{kc}{en_e}\nabla n_e\times\nabla T_e + \frac{c}{e}\nabla\times\frac{\mathbf{R}_T}{n_e} - c\,\nabla\times(\eta\mathbf{j}) - \frac{1}{e}\nabla\times\frac{\mathbf{j}\times\mathbf{B}}{n_e}\,. \tag{60}$$

The quantities and expressions are: ρ — mass density, $\mathbf{w}(v,0,u)$ — fluid velocity, T_i — ion temperature, T_e — electron temperature, $\mathbf{B}(0,B_\phi,0)$ — magnetic field, z — degree of ionization, $\mathbf{j}(0,0,j_z)$ — current density, $\boldsymbol{\sigma}$ — stress tensor, η — resistivity, c_v — specific heat, τ_{ei} — electron-ion relaxation time, Q_i — energy loss, n_e — electron density, R_T — thermoelectric force, c — speed of light, e — electron charge and k — Boltzmann constant.

The stress tensor $\boldsymbol{\sigma}$ can be specified as follows:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^e + \boldsymbol{\sigma}^i, \tag{61}$$

$$\sigma^e_{ik} = -p_e\delta_{ik}, \tag{62}$$

$$\sigma^i_{ik} = -p_i\delta_{ik} + 2\mu S_{ik}, \tag{63}$$

$$S_{ik} = \dot\varepsilon_{ik} - \frac{1}{3}\dot\varepsilon_{ll}\delta_{ik}, \tag{64}$$

$$\dot{\boldsymbol{\varepsilon}} = \begin{bmatrix} \frac{\partial u}{\partial r} & 0 & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) \\ 0 & \frac{u}{r} & 0 \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) & 0 & \frac{\partial v}{\partial z} \end{bmatrix}, \tag{65}$$

where μ — coefficient of viscosity, p_e — electron pressure and p_i — ion pressure.

The current density $\mathbf{j}$ is related to magnetic field intensity by the Ampere's law

$$\mathbf{j} = \frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B}. \tag{66}$$

The quantity c_v is defined as

$$c_v = \frac{k}{(\gamma-1)m_j}. \tag{67}$$

It has been assumed for the purpose of the numerical experiment that $\gamma = \frac{5}{3}$ and $m_j = 3.35\times10^{-23}\,\mathrm{g}$ (mass of the neon ion).

The equation of state has been assumed in the form

$$p = p_e + p_i, \tag{68}$$

$$p_e = z(\gamma-1)c_v\rho T_e, \tag{69}$$

$$p_i = (\gamma-1)c_v\rho T_i. \tag{70}$$

The electron density has been evaluated from the formula

$$n_e = z\frac{\rho}{m_j} \tag{71}$$

and the degree of ionization z by means of the collision-radiative model

$$\frac{\mathrm{d}z}{\mathrm{d}t} = z(f_i - f_j - f_t). \tag{72}$$

The rates of the ionization process f_i and those of the recombination process f_j and f_t were obtained from the relation

$$Q_i = \frac{1}{m_j}\int_1^z J(z)\,\mathrm{d}z \tag{73}$$

where $J(z)$ is the average ionization potential. The expression for the thermoelectric force has the form

$$\mathbf{R}_t = -kn_e\left[\beta_\perp\boldsymbol{\nabla}_\perp T_e + \beta_\wedge(\mathbf{i}\times\boldsymbol{\nabla}T_e)\right] \tag{74}$$

where:

$$\mathbf{i} = \frac{\mathbf{B}}{|\mathbf{B}|} \tag{75}$$

and $\beta_\perp$, $\beta_\wedge$ are thermoelectric coefficients.

The expressions of W_i and W_e involved in the energy equations describe the transport processes of heat and radiation,

$$W_i = -\nabla \cdot \mathbf{h}^i, \tag{76}$$
$$W_e = -\nabla \cdot \mathbf{h}^e - \nabla \cdot \mathbf{h}^p. \tag{77}$$

The ion heat flux in the magnetized plasma has the form:

$$\mathbf{h}^i = -\kappa_\perp^i \nabla_\perp T_i - \kappa_\wedge^i (\mathbf{i} \times \nabla T_i). \tag{78}$$

The components of electron heat flux $\mathbf{h}^e$ and radiation flux $\mathbf{h}^p$ are thus expressed by the formulae:

$$\mathbf{h}^e = \mathbf{h}^T + \mathbf{h}^w, \tag{79}$$
$$\mathbf{h}^T = -\kappa_\perp^e \nabla_\perp T_e - \kappa_\wedge^e (\mathbf{i} \times \nabla T_e), \tag{80}$$
$$\mathbf{h}^w = -\frac{kT_e}{e}\left[\beta_\perp \mathbf{j} + \beta_\wedge (\mathbf{i} \times \mathbf{j})\right], \tag{81}$$
$$\mathbf{h}^p = -\kappa^p \nabla_\perp T_e, \tag{82}$$

where

$$\kappa^p = \frac{16\sigma_S l_R T_e^3}{3}, \tag{83}$$

σ_S — Stefan-Boltzmann constant and l_R is the Rosseland length.

The boundary conditions were as follows

$\sigma_n = \sigma_\tau = 0$	on the surfaces,	(84)
$v = 0, \quad \sigma_\tau = 0$	at the electrodes,	(85)
$B_\phi = \frac{2J(t)}{cR}$	on the free surface of the pinch,	(86)
$E_r^T = 0$	at the electrodes,	(87)
$h_n^i = 0, \quad h_n^e = \frac{cU^p}{2}$	on the free surface,	(88)
$T_e = T_i = T_0$	on the electrode surfaces,	(89)

where h_n^i, h_n^e — the flux components normal to the surface, T_0 — initial electrode temperature and $R = R(z,t)$ — radius of the pinch surface. E_r^T — radial component of total electric field — is expressed as follows,

$$E_r^T = -cE_r + \frac{E_z B_\phi}{e n_e \eta} + \frac{kc}{e}\left[\frac{1}{n_e}\frac{\partial (n_e T_e)}{\partial r} + \beta_\perp \frac{\partial T_e}{\partial r} + \beta_\wedge \frac{\partial T_e}{\partial z}\right], \qquad E_r = \eta j_r, \qquad E_z = \eta j_z. \tag{90}$$

The radiation density has the form

$$U^p = \frac{4\sigma_S T_e^4}{c}. \tag{91}$$

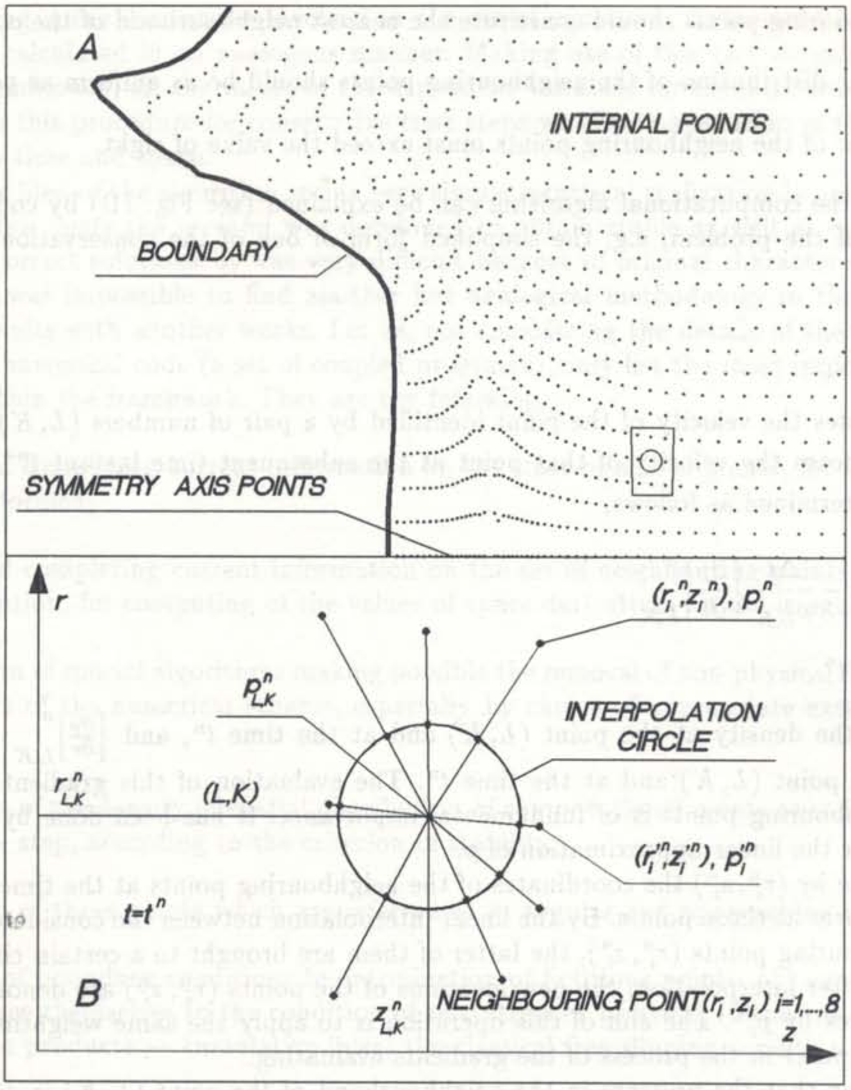

Fig. 1. The idea of the computational algorithm of the method of free points (particles)

3. Numerical methods

The sets of differential equations and relations describing the properties of the materials, presented in a foregoing Section with the relevant initial-boundary conditions have been solved numerically by means of what is referred to as a "method of free particles", sometimes also called Dyachenko's "method of free points" [6] and has been discussed in brief in the works [2, 4] and, in greater detail, in [3]. This method may be used for solving various non-stationary, two-dimensional boundary value problems of a continuum. The propagation of the shock wave front is modelled with quadratic Richtmyer-von Neumann artificial viscosity.

The object of our study is the system of material points moving together with the body in the course of its deformation, see Fig. 1A. At the initial instant of time, these points are located so that they form sufficiently dense mesh covering the object to be studied, in agreement with its geometry. The values of the dependent variables are prescribed at each point. The motion of the points is followed in the course of solving the problem, new values of the parameters being calculated. The motion and the parameters at each point are determined from the equations of the problem, taking into account the parameters at some neighbouring points which are the nearest ones to the point considered. The selection of the neighbouring points should be done according to the following principles:

(a) the neighbouring points should constitute the nearest neighbourhood of the considered point,

(b) the angular distribution of the neighbouring points should be as uniform as possible,

(c) the number of the neighbouring points must exceed the value of eight.

The idea of the computational algorithm can be explained (see Fig. 1B) by considering one of the equations of the problem, e.g. the simplified form of one of the conservation of momentum equations

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial p}{\partial r}. \tag{92}$$

If $u^n_{L,K}$ denotes the velocity of the point identified by a pair of numbers (L, K) at the time t^n and if $u^{n+1}_{L,K}$ denotes the velocity of that point at the subsequent time instant t^{n+1}, the quantity $u^{n+1}_{L,K}$ can be determined as follows,

$$u^{n+1}_{L,K} = u^n_{L,K} - \frac{\Delta t}{\rho^n_{L,K}}\left[\frac{\partial p}{\partial r}\right]^n_{L,K}, \tag{93}$$

$$\Delta t = t^{n+1} - t^n, \tag{94}$$

where $\rho^n_{L,K}$ is the density at the point (L, K) and at the time t^n, and $\left[\frac{\partial p}{\partial r}\right]^n_{L,K}$ is the pressure gradient at the point (L, K) and at the time t^n. The evaluation of this gradient in the moving meshes of neighbouring points is of fundamental importance. It has been done by the method of least squares for the linear approximation of p.

Let us denote by (r^n_i, z^n_i) the coordinates of the neighbouring points at the time instant t^n and by p^n_i the pressures at those points. By the linear interpolation between the considered point (L, K) and the neighbouring points (r^n_i, z^n_i), the latter of them are brought to a certain circle around the point (L, K). After interpolation, the new locations of the points (r^n_i, z^n_i) are denoted by (r'^n_i, z'^n_i) and the pressures by p'^n_i. The aim of this operation is to apply the same weighting procedure to each particular point in the process of the gradients evaluating.

Let us assume that the pressure in the neighbourhood of the point (L, K) is described by the linear function

$$p^n(r, z) = p^n_{L,K} + a(r - r^n_{L,K}) + b(z - z^n_{L,K}) \tag{95}$$

where $r^n_{L,K}$, $z^n_{L,K}$ and $p^n_{L,K}$ are coordinates of the point (L, K) and the pressure, respectively. The coefficients a and b can be calculated by the method of least squares,

$$\xi(a, b) = \sum_{i=1}^{N} \left[p^n(r'^n_i, z'^n_i) - p'^n_i\right]^2, \tag{96}$$

$$\frac{\partial \xi}{\partial a} = 0, \tag{97}$$

$$\frac{\partial \xi}{\partial b} = 0. \tag{98}$$

These formulae give the values of the coefficients a and b which are also the gradients

$$a = \left[\frac{\partial p}{\partial r}\right]^n_{L,K}, \tag{99}$$

$$b = \left[\frac{\partial p}{\partial z}\right]^n_{L,K}. \tag{100}$$

All the remaining space derivatives involved in the set of equations, formulated in the preceding Section can be calculated in an analogous manner. Making use of this fact we can find, from the formulae just mentioned, all the values of the dependent variables for the next time-level t^{n+1}.

By repeating this procedure for consecutive time steps we obtain a solution of the problem in a discrete form in time and space.

Although the idea of the algorithm seems very simple, practical realization is quite complicated. Over the past few years the method was developed to obtain stable as well as qualitatively and quantitatively correct solutions. It was very difficult because of original character of this method. For example it was impossible to find another but analogical methodology in the literature and compare our results with another works. Let us, not considering the details of the construction of an appropriate numerical code (a set of coupled programs), only list the most important problems to be solved within the framework. They are the following:

(a) storing and completing current information on the state of all the points, for which computation is performed,

(b) storing and completing current information on the set of neighbouring points, for each point of computation, for computing of the values of space derivatives on the irregular grid,

(c) introduction of special algorithms making possible the removal of non-physical fluctuations and instabilities of the numerical scheme, especially by choice of appropriate external numerical diffusion,

(d) adjustment of the density of spatial distribution of computational points as well as adjustment of the time step, according to the criterion of stability,

(e) elimination of these points which are referred to as angular and approaching instabilities,

(f) modelling of boundary conditions by introduction of fictitious points: (1) on all free surfaces which reduce themselves to the condition of zero stress or pressure; (2) on the contact surface (detonation products — cumulative liner) the classical free slipping condition,

(g) modelling of the condition on the symmetry axis by introduction of fictitious points,

(h) construction of special algorithms for computing rapid processes (in time step scale) such as: stress relaxation, growth of cracks etc., and elimination of fragmented parts of solids,

(i) modelling of processes of generation and propagation of shock waves.

4. Examples of results of computer simulation studies

The scope of the present paper being limited, we shall only present some examples of the results to illustrate the possibilities of computer simulation of such problems by means of the method.

Figure 2 shows, as an example, the result of computer simulation study of the shaped charge jet formation and the penetration process of a steel armour plate. Consecutive frames illustrate the process of development of the detonation, deformation of the conical copper liner and the aluminium casing as well as generation of the shaped charge jet.

Figure 3 shows the results of simulation of the explosive forming of projectile.

Figure 4 illustrates the process of modelling of the armour plate penetration.

Figures 5–8 regard modelling of the hypervelocity asteroids collisions with the Earth.

Figure 5 presents several snap shots of the impact of the 10 km diameter asteroid on the ocean and the multilayered Earth's litosphere.

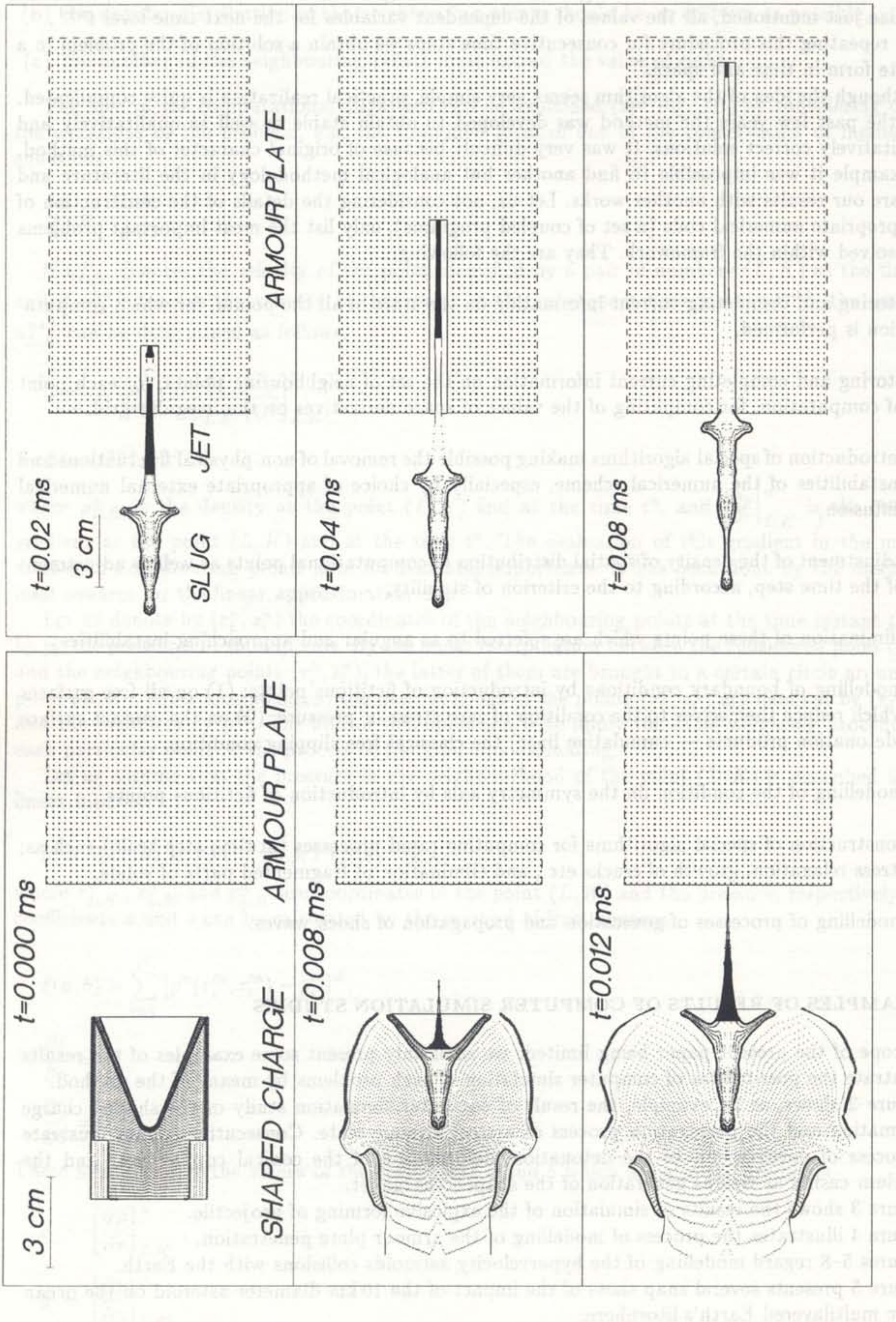

Fig. 2. Left: Shaped charge jet formation. Consecutive frames represent the detonation process, deformation of a conical copper liner and aluminium case as well as generation of the jet. Right: Armour penetration by the jet

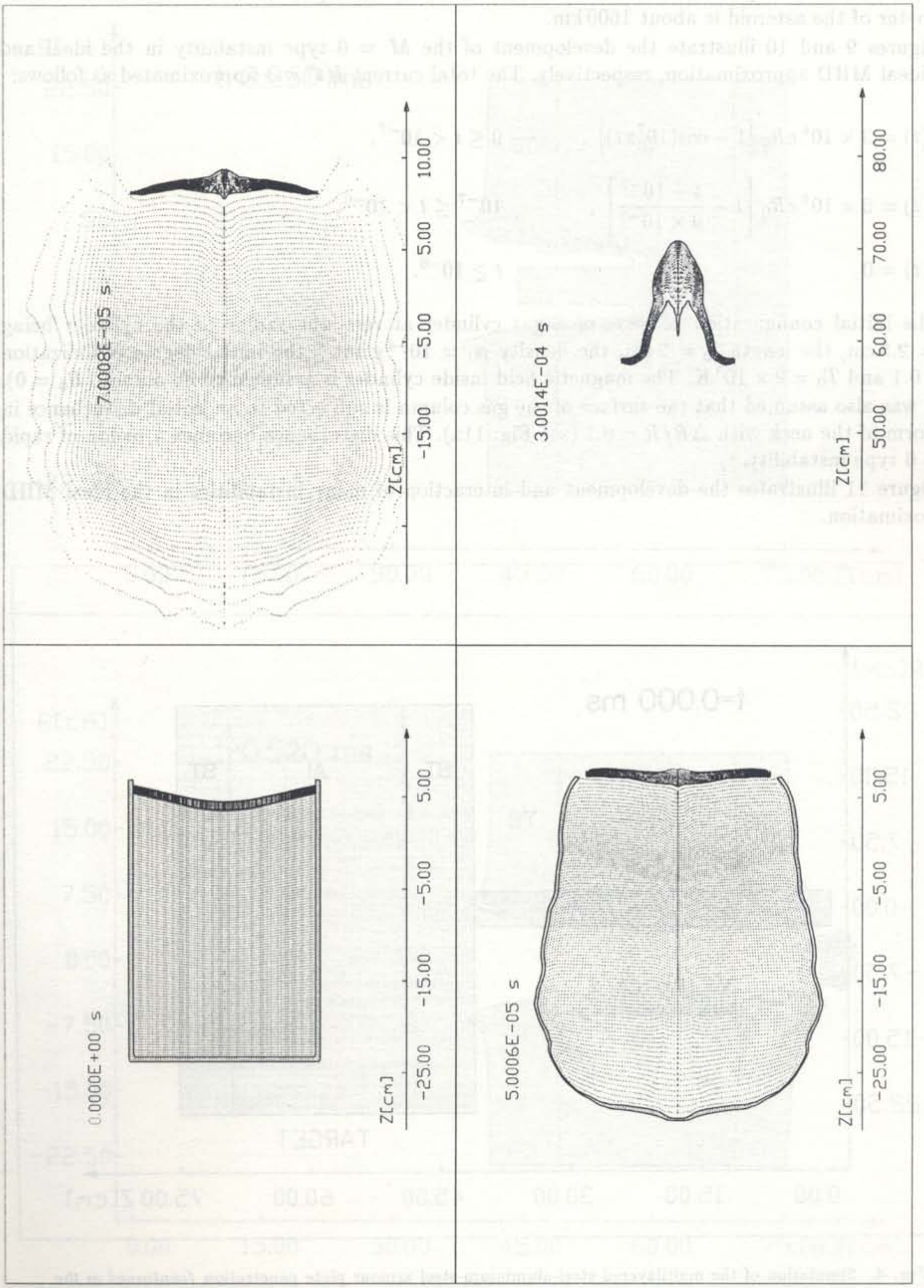

Fig. 3. Simulation of the explosive forming of projectile

On Figs. 6 and 7 we demonstrate results of the simulation of the shock wave propagation through the Earth and velocity vector of the Earth surface after collision with an asteroid.

Figure 8 shows snap shots of the simulation of the sub-giant impact of the asteroid on the Earth. Diameter of the asteroid is about 1600 km.

Figures 9 and 10 illustrate the development of the $M = 0$ type instability in the ideal and non-ideal MHD approximation, respectively. The total current $I(t)$ was approximated as follows:

$$I(t) = 3 \times 10^4\, cR_0 \left[1 - \cos(10^7 \pi t)\right], \qquad 0 \le t < 10^{-7},$$

$$I(t) = 6 \times 10^4\, cR_0 \left[1 - \frac{t - 10^{-7}}{9 \times 10^{-7}}\right], \qquad 10^{-7} \le t < 10^{-6},$$

$$I(t) = 0 \qquad t \ge 10^{-6}.$$

The initial configuration consists of a gas cylinder at rest, the radius of the cylinder being $R_0 = 2.5\,\mathrm{cm}$, the length $l_0 = 2\,\mathrm{cm}$, the density $\rho_0 = 10^{-4}\,\mathrm{gcm}^{-3}$, the initial degree of ionization $z_0 = 0.1$ and $T_0 = 2 \times 10^3\,\mathrm{K}$. The magnetic field inside cylinder is assumed to be absent ($B_\phi = 0$).

It was also assumed that the surface of the gas column is subjected to an initial disturbance in the form of the neck with $\Delta R/R = 0.1$ (see Fig. 11a). This disturbance becomes a region of rapid $M = 0$ type instability.

Figure 11 illustrates the development and interaction of many instabilities in the ideal MHD approximation.

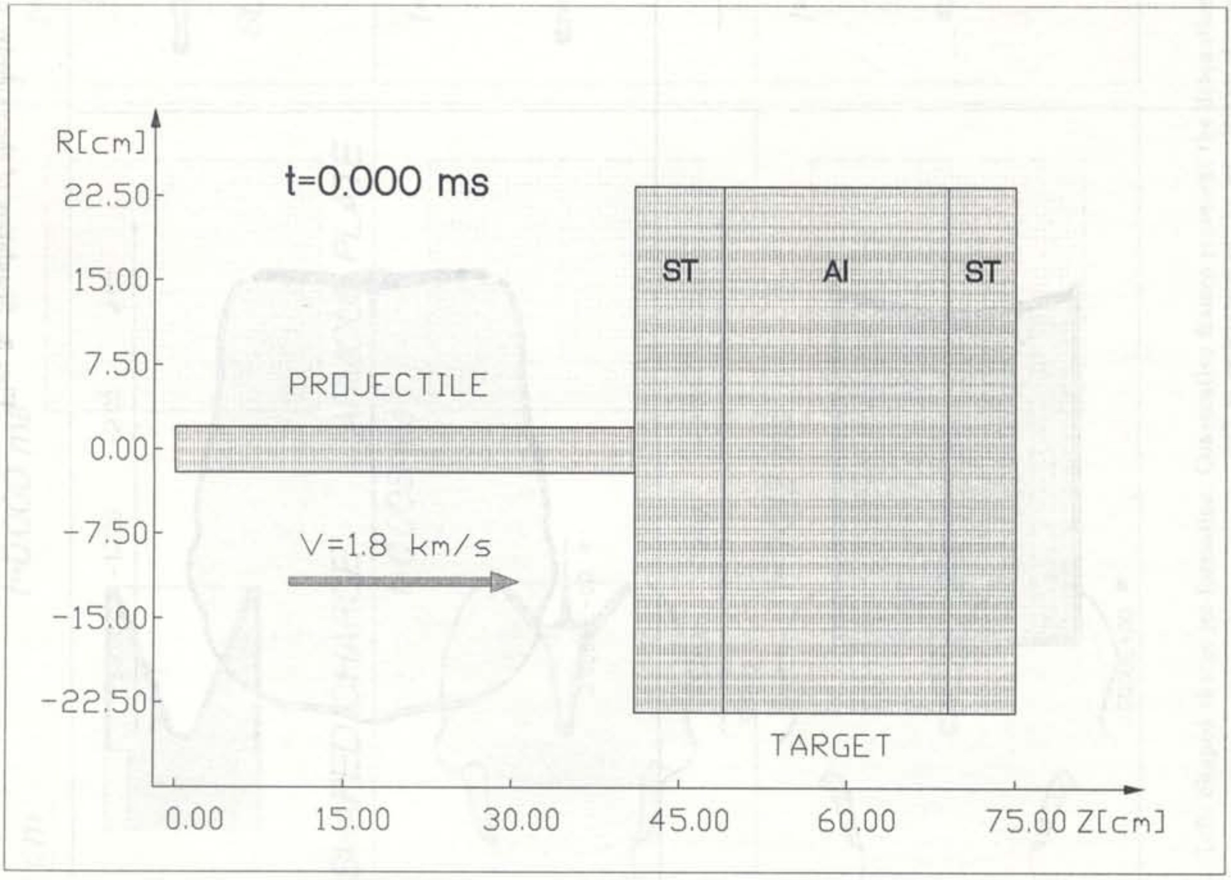

Fig. 4. Simulation of the multilayered steel-aluminium-steel armour plate penetration *(continued in the next page)*

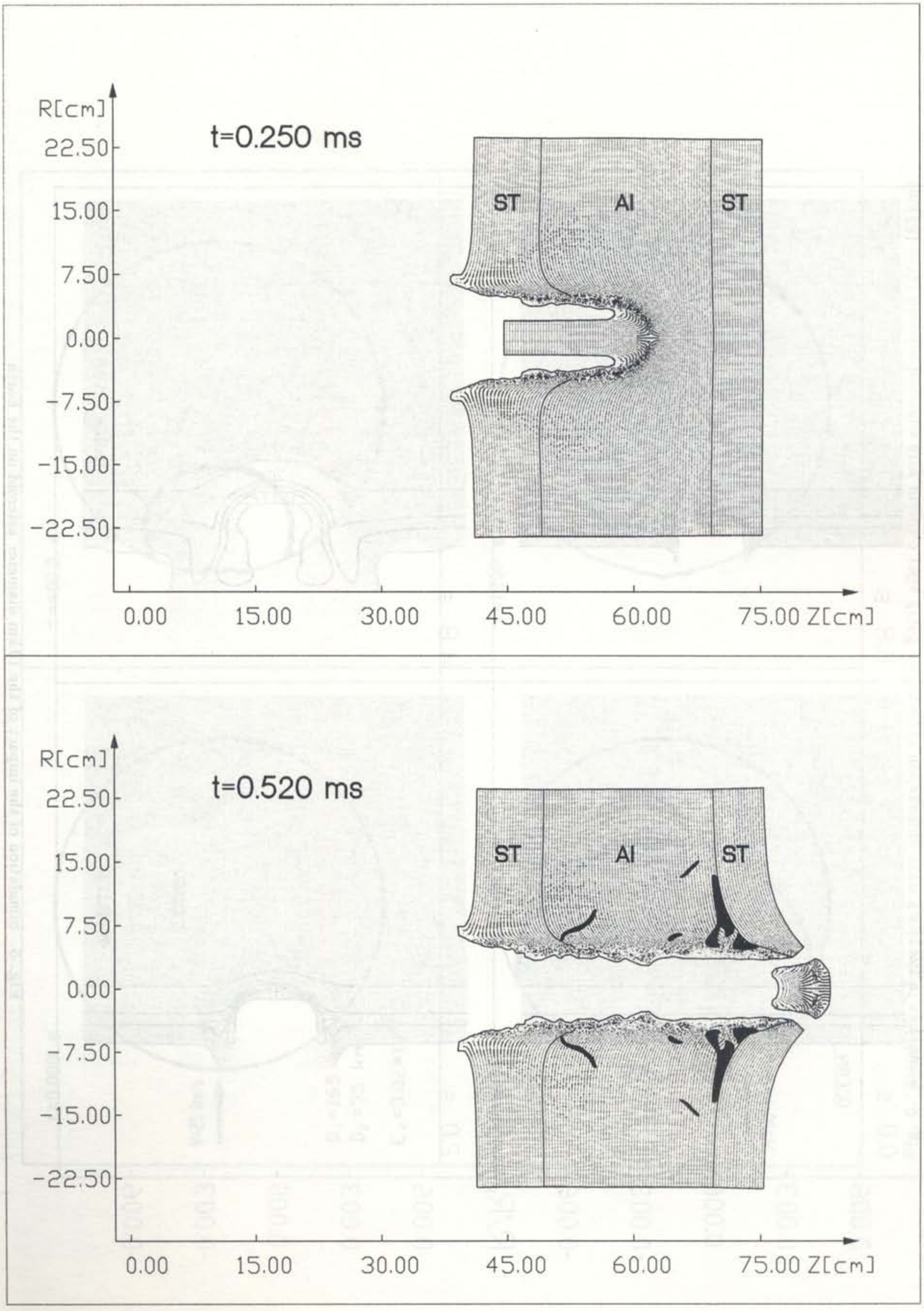

Fig. 4. *(continued)*

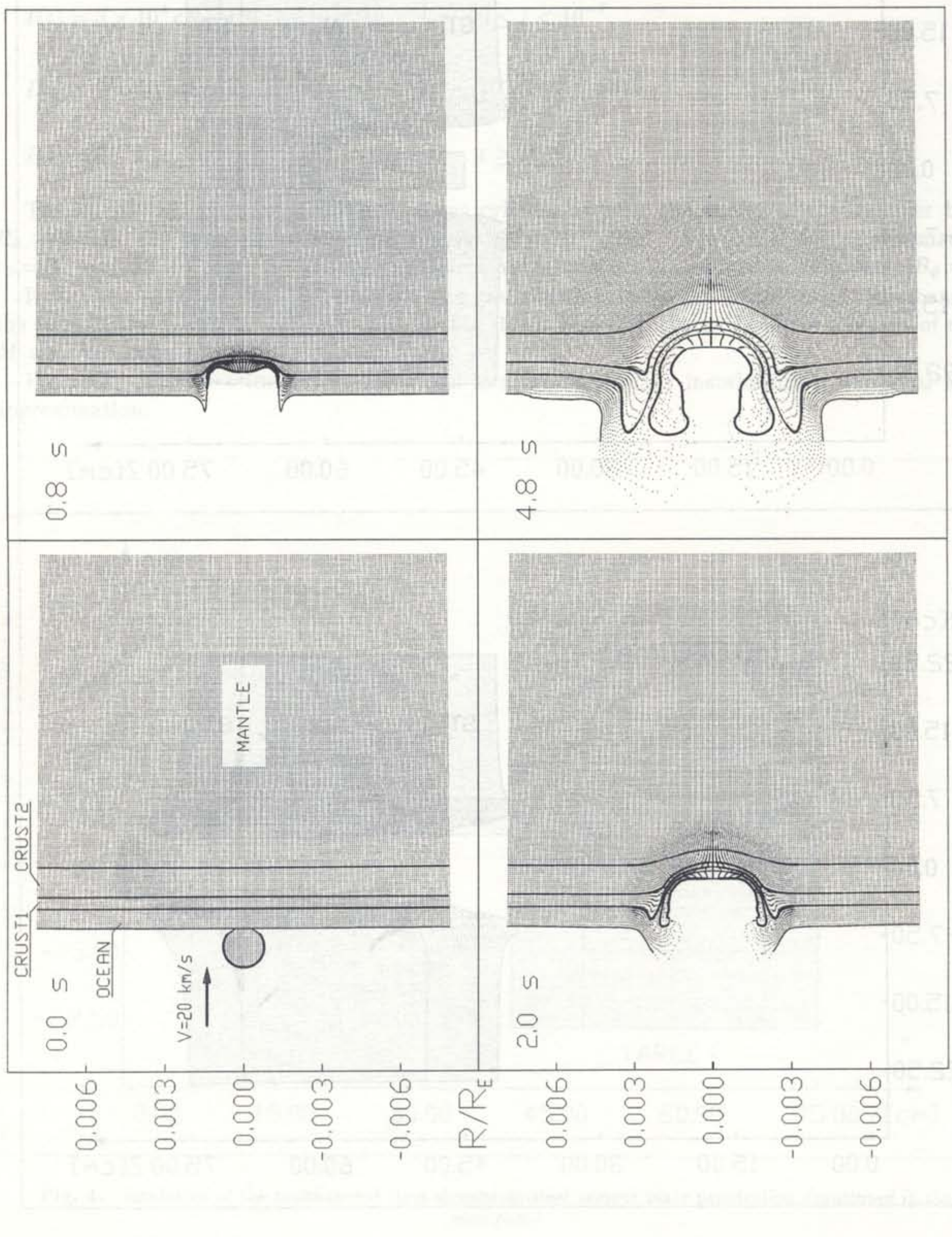

Fig. 5. Simulation of the impact of the 10 km diameter asteroid on the Earth

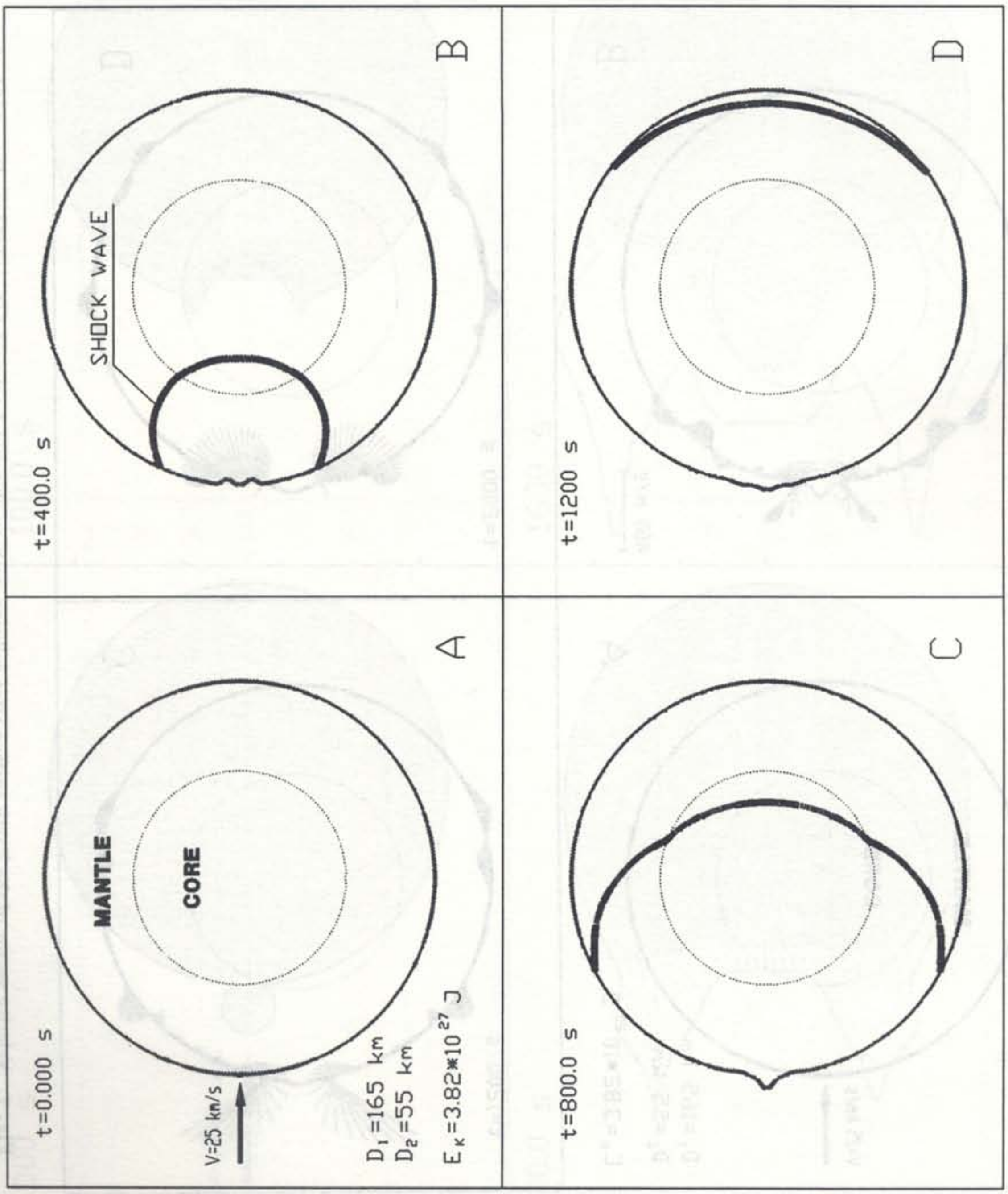

Fig. 6. Simulation of the shock wave propagation through the Earth after collision with spherical asteroid ($D = 165$ km).

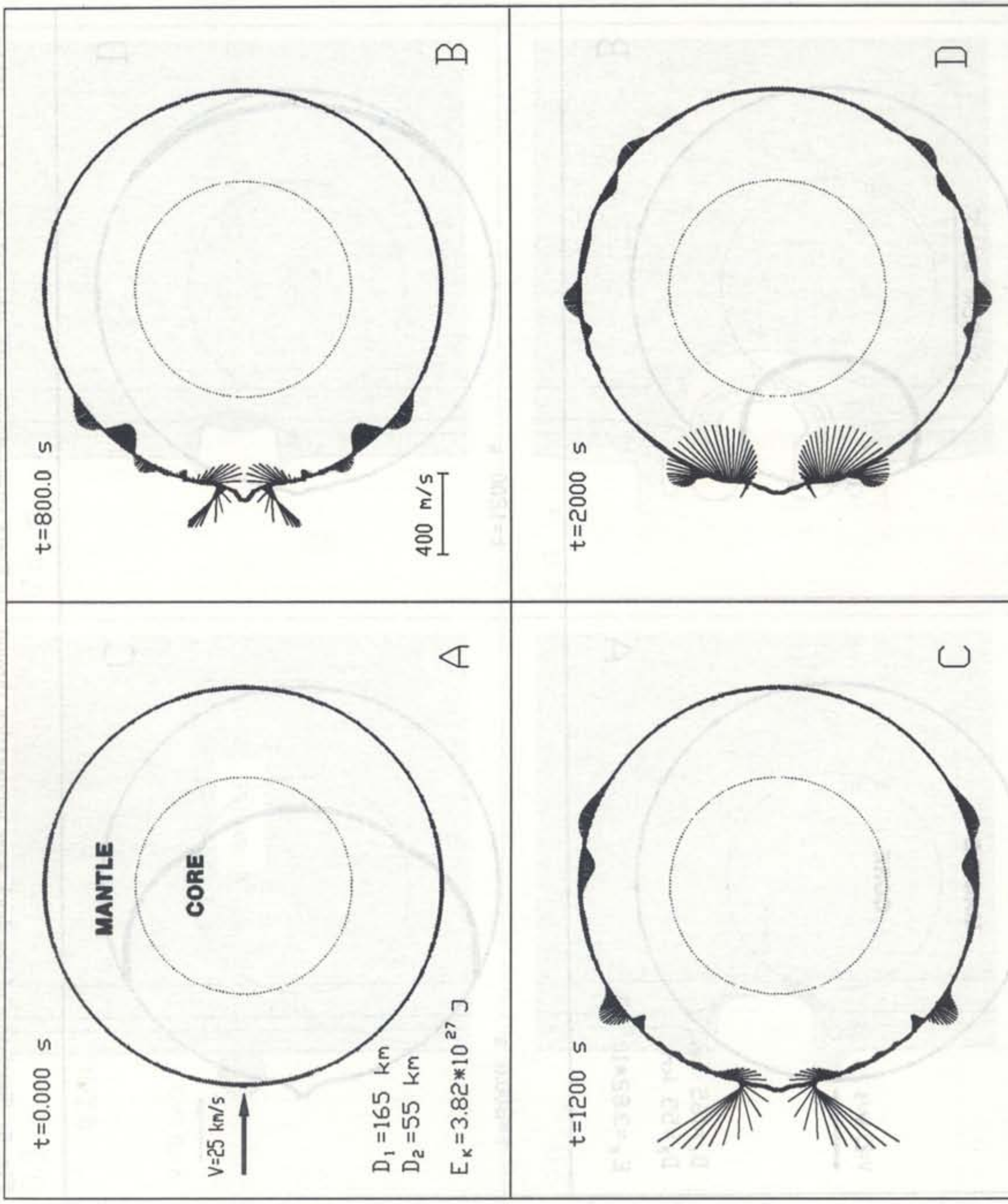

Fig. 7. Velocity vector of the Earth surface after collision with an ellipsoidal asteroid ($D_1 = 165$ km, $D_2 = 110$ km)

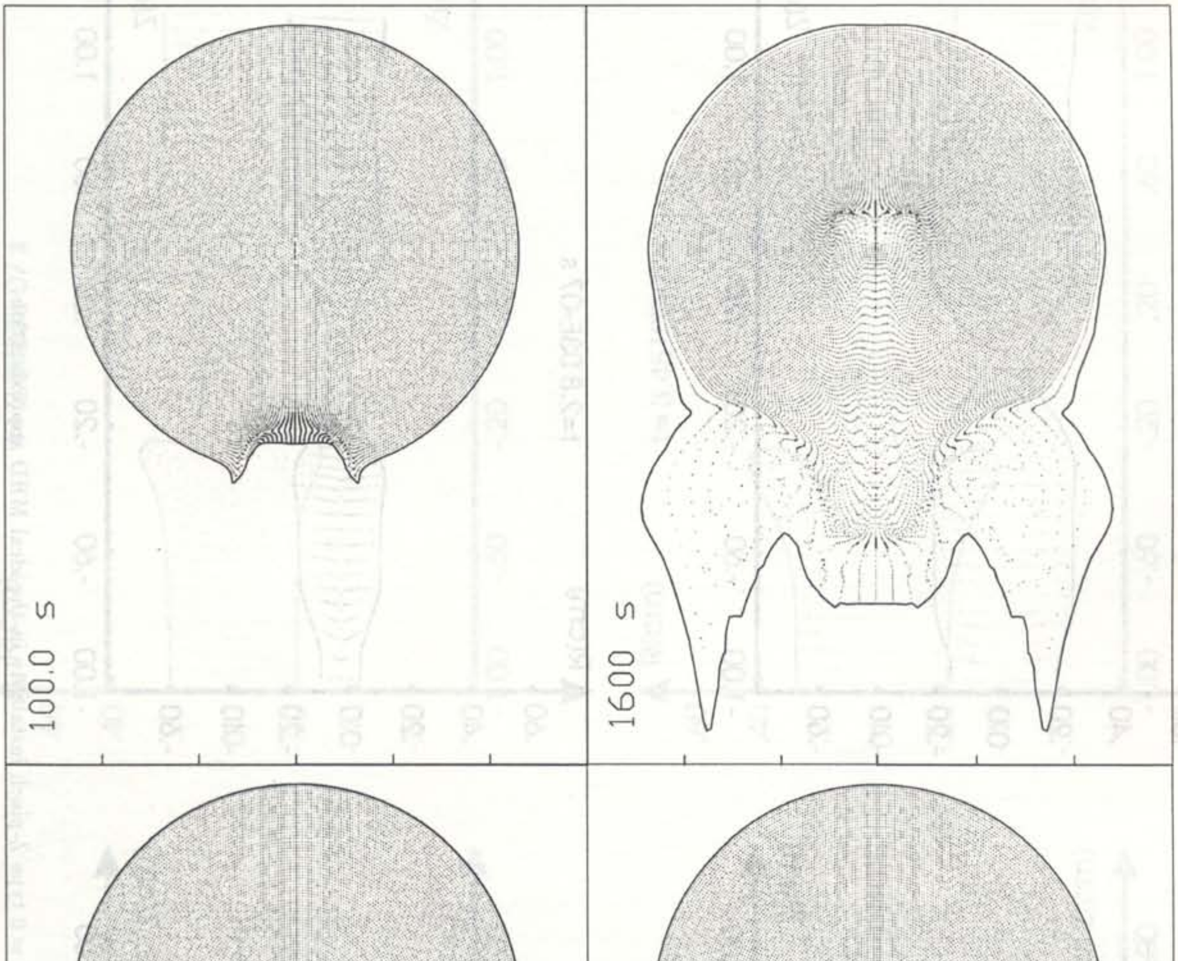

Fig. 8. Snap shots of the simulation of the sub-giant impact of the asteroid on the Earth. Diameter of the asteroid is about 1600 km

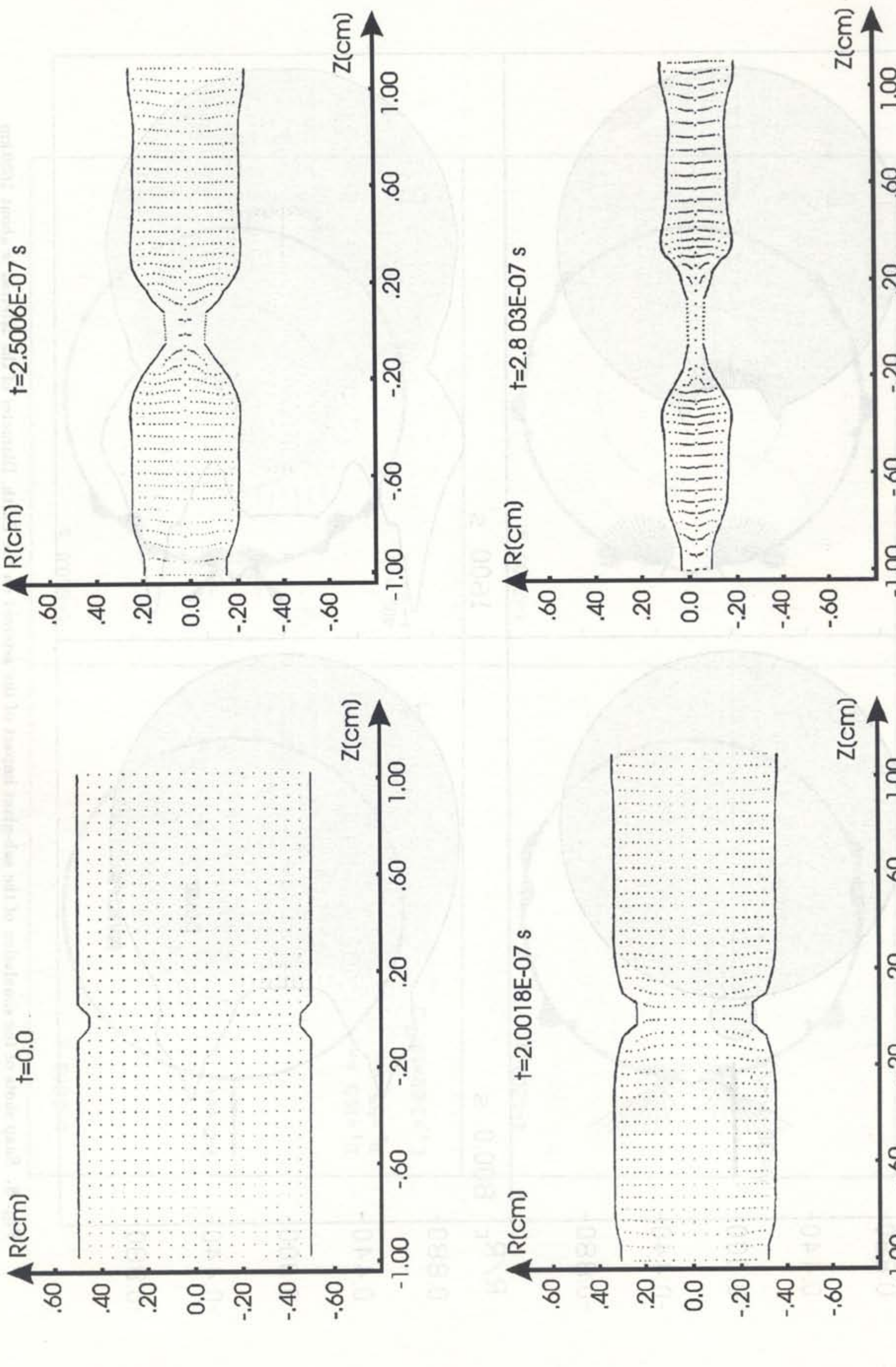

Fig. 9. Development of the $M = 0$ type Z-pinch instability in the ideal MHD approximation

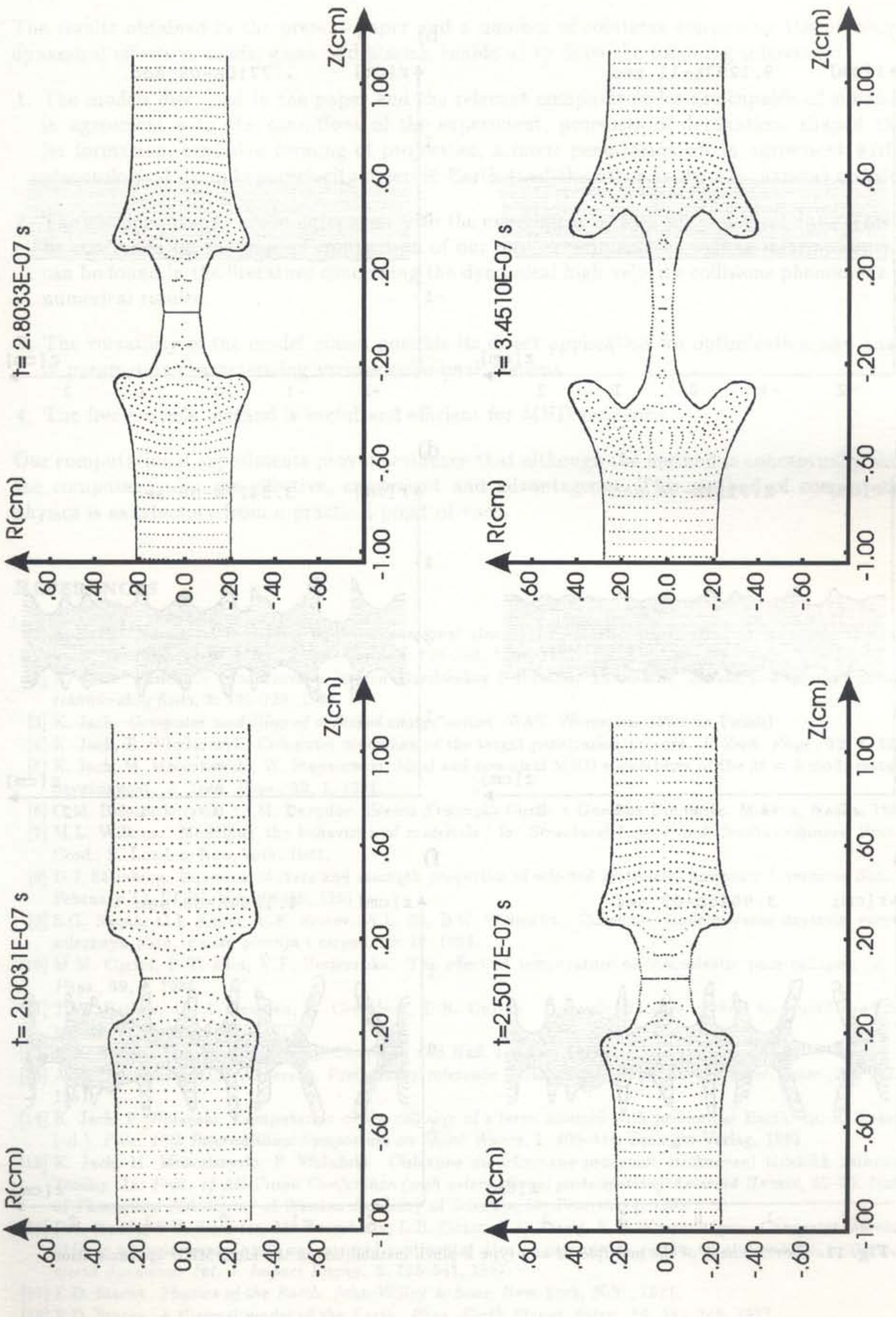

Fig. 10. Development of the $M = 0$ type Z-pinch instability in the non-ideal MHD approximation (initial state — the same as in the previous case)

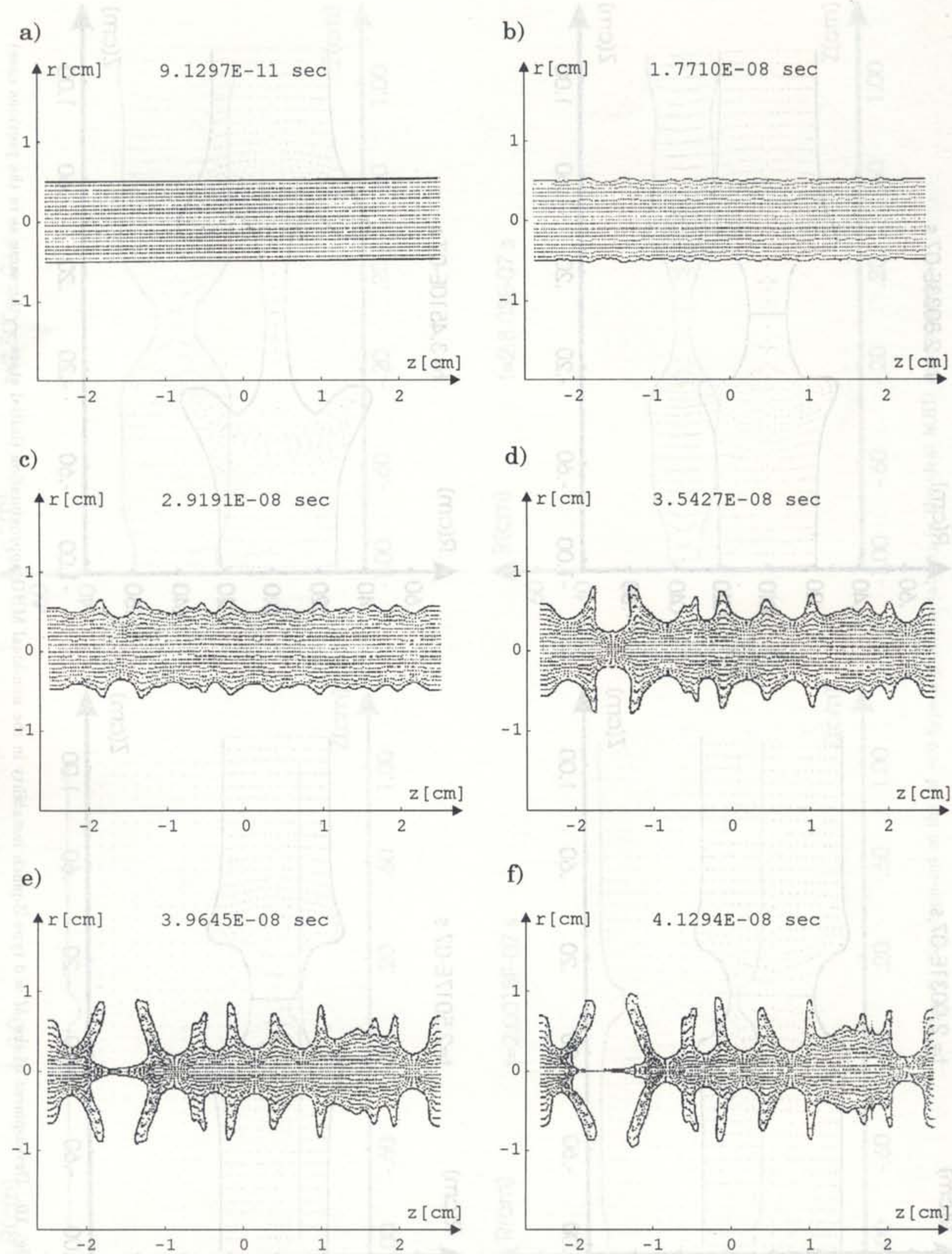

Fig. 11. Development of the multiple $M = 0$ type Z-pinch instabilities in the ideal MHD approximation

5. Conclusions

The results obtained in the present paper and a number of solutions concerning the problems of dynamical effects in solids, gases and plasma enable us to draw the following inferences:

1. The models discussed in the paper and the relevant computer codes are capable of simulating, in agreement with the conditions of the experiment, processes of detonation, shaped charge jet formation, explosive forming of projectiles, armour penetration or, in agreement with the planetological data, hypervelocity asteroid-Earth (and the other Earth-like planets) collisions.

2. The obtained results are in agreement with the experiment or with seismological data. This may be concluded on the base of comparison of our own experiments as well as information which can be found in the literature concerning the dynamical high-velocity collisions phenomena with numerical results.

3. The versatility of the model makes possible its direct application for optimization and analysis of parameters characterizing various collisional systems.

4. The free particle method is useful and efficient for MHD problems.

Our computational experiments provide evidence that although the method is conceptually simple, the computer codes are effective, convenient and advantageous. This method of computational physics is satisfactory from a practical point of view.

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