Computer Assisted Mechanics and Engineering Sciences, 7: 103–116, 2000.

A polyoptimal catalogue of rolled profiles for a given class of spatial trusses

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(Received August 5, 1999)

The purpose of this polyoptimization was to find optimal tubular cross section catalogues for the specified spatial trusses. Five different spans of trusses have been analyzed, starting from 24 × 24 m ending at 72 × 72 m, trusses supported at every other perimeter node, with loading conditions typical for roofs. Four decision variables were taken under consideration, i.e. the catalogue size t, its arrangement T in a metallurgical catalogue T_M, minimal and maximal diameters of tubular elements $D_{\min}$ and $D_{\max}$. Two objective functions: truss mass and manufacturability of particular solutions were evaluated. For that purpose, some design, technological, and computational constraints were taken into account. For the specified spans L, different catalogues of cross sections were defined by TOPSIS method.

1. Introduction

The spatial truss structure made of tubular bars connected at nodes is defined by its topology. Even if the connections of bars are not exactly hinged, for the purpose of design it may be assumed as a model of spatial truss. Spatial trusses are widely used as roof coverings, ceilings and walls in different kinds of buildings [2, 3, 9].

One of the most demanding problem for that kind of structures is a selection of cross sections catalogue. Without effective optimization algorithms, that problem can be solved only approximately by skilled and experienced designer. With large number of structural elements and decision variables, algorithmic formulation of the problem leads usually to so called *NP-hard* combinatory problems. Parameters describing type and cross sections of bars, size of a catalogue of cross sections and a system of zones with the same stiffness are usually used as decision variables in optimization problems of spatial truss structures [4–8, 10–12, 14].

The catalogue of cross sections of spatial truss structure elements should be selected from a metallurgical catalogue T_M. Truss mass and manufacturability of particular solutions depends on the catalogue size t and its positioning T in a metallurgical catalogue T_M. The cross section catalogue can be specified as a result of solving a polyoptimization problem. Two opposite goals seem to be of importance here. First, the catalogue size t should be large enough to utilize all the cross sections to a maximal degree, and second, the number of cross sections should be minimal for the purpose of reducing assembly costs and improving manufacturability defined below by function (4). The cross section unification leads to a reduction of the number of node types, reduction of assembly costs and transportation.

The main purpose of this paper is to present a method of polyoptimal selection of the catalogue size t and its positioning T in a metallurgical catalogue T_M, for a series of spatial trusses of spans from $L \times L = 24 \times 24$ m to $L \times L = 72 \times 72$ m. It is done for varied constraints on minimal $D_{\min}$ and maximal $D_{\max}$ cross section diameters. The truss mass and manufacturability of particular solutions are also being evaluated.

2. The formulation of the problem

Most often, the spatial trusses are made of steel tubular cross sections [9]. The part of a metallurgical catalogue of such cross sections [1] is shown in Fig. I. The external diameter D ranging from 31.8 to 508 mm and thickness g of no more than 20 mm is taken into consideration.

In this paper four decision variables are used:

$$\mathbf{x} = [x_1\, ,\, x_2\, ,\, x_3\, ,\, x_4]^T, \tag{1}$$

where:

$x_1 \equiv t$ – catalogue size, $t \in \{25, 13, 9, 7, 5\}$, see Table 1; the sequential derivative catalogues are created from the previous one by erasing every second cross section,

$x_2 \equiv T$ – positioning of the 25-element catalogue in a metallurgical catalogue T_M ,

$x_3 \equiv D_{\min}$ – the minimal external diameter of a tubular cross section,

$x_4 \equiv D_{\max}$ – the maximal external diameter of a tubular cross section.

Table 1. Numbers of base and derivative catalogues

t	Numbers of bar cross sections in a catalogue with the number of bars t																								
25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
13	1		2		3		4		5		6		7		8		9		10		11		12		13
9	1			2			3			4			5			6			7			8			9
7	1				2				3				4				5				6				7
5	1						2						3						4						5

The arrangement of catalogue T of cross sections in a metallurgical catalogue T_M can be selected deterministically or randomly. Twenty five elements taken randomly from 296 give $296!/(25! \cdot (296 - 25)!)$ possibilities of positioning for these elements. Part of these catalogues so selected do not comply with the design constraints of a problem. The checking process necessary for filtering them out is time-consuming, for it implies, several iteration steps for the selection of all truss elements. That is why in this paper a deterministic procedure has been employed. The previous works [6–8, 10–12] have also been taken into account. They show that the cross sections should be situated in the metallurgical catalogue along the lower limits of thickness g, see Fig. I. Figure I shows all the cross sections that have been analyzed for different positionings T and different spans L. The restriction on the decision variable x_4 (representing the maximal external diameter $D_{\max}$) implies significant increase of g. So for spans $L \geq 36$ m, the basic catalogue enters the internal part of the metallurgical catalogue T_M (comes closer to the upper limit for g). The limit on the minimal diameter $x_3 = D_{\min}$ improves the so called manufacturability of the solution. At the same node of the truss, bars with cross sections ranging from $D_{\min}$ to $D_{\max}$ can meet. It seems useful that the ratio $D_{\max}/D_{\min}$ be as small as possible.

The basic catalogue t_B contains 25 elements. That number of elements allows to keep the same first and last elements in derivative catalogues T. The way of creating all the derivative catalogues from the basic one is shown in Table 1.

The cross sections catalogues, from which all the roof covers have been created, should fulfil two objective functions, i.e., the vector function:

$$\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), \ldots, f_j(\mathbf{x}), \ldots, f_J(\mathbf{x})]^T = [f_1(\mathbf{x}), f_2(\mathbf{x})]^T, \tag{2}$$

where: $f_1(\mathbf{x})$ is a mass of the truss for 1 m^2 of horizontal projection of covered surface and $f_2(\mathbf{x})$ is a so called manufacturability of the solution. The latter takes care of all other decision variables of the problem.

The mass of the truss is automatically evaluated by the OPTYTRUSS system. The cross sections are selected from the given catalogue by iteration using displacement method in the TRUSS program. During the calculations, the least advantageous loading possible are applied to the structure. The truss mass function is

$$f_1(\mathbf{x}) = \frac{\gamma m_n}{L^2} \sum_{i=1}^{I(L)} A_i(\mathbf{x})\, l_i(L) \tag{3}$$

where:

$\gamma = 7850\,\text{kg/m}^3$ – the mass of steel per 1 cubic meter,

$m_n = 1.05$ – the coefficient increasing the mass of truss bars coming from the additional mass of the nodes,

A_i , l_i – the cross section area and the length of the i-th bar,

$I(L)$ – the number of bars in the given truss, $I(L) = 8p^2 \in \{800, 1152, 1568, 2048, 2592\}$ (where $p = L/a$, L is a truss span and a is a distance between truss nodes).

The manufacturability of the catalogue can be evaluated as follows

$$f_2(\mathbf{x}) = [1 + c_1 (1 + c_2 - 0.001\, c_2 I(L))\, t] \left[1 + c_3 \frac{D_{\max}}{D_{\min}} \left(\frac{1}{t}\right)^{\frac{1}{t-1}}\right] \left[1 + c_4 \frac{t^{1.1}}{t_D} \left(\frac{A_{\max}}{A_{\min}}\right)^{\frac{1}{t-1}}\right] \tag{4}$$

where:

c_1 – coefficient of the size of the catalogue, showing the increase of assembly cost when the catalogue number increases two times $c_1 \in \langle 0.005, 0.015\rangle$ [11]; accepted $c_1 = 0.01$;

c_2 – coefficient of the number of bars in the truss, showing the decrease of assembly cost when the number of elements of the truss increases two times, $c_2 \in \langle 0.05, 0.15\rangle$ [11]; accepted $c_2 = 0.1$;

c_3 – coefficient of compactness of the catalogue $c_3 \in \langle 0.01, 0.02\rangle$; accepted $c_3 = 0.015$;

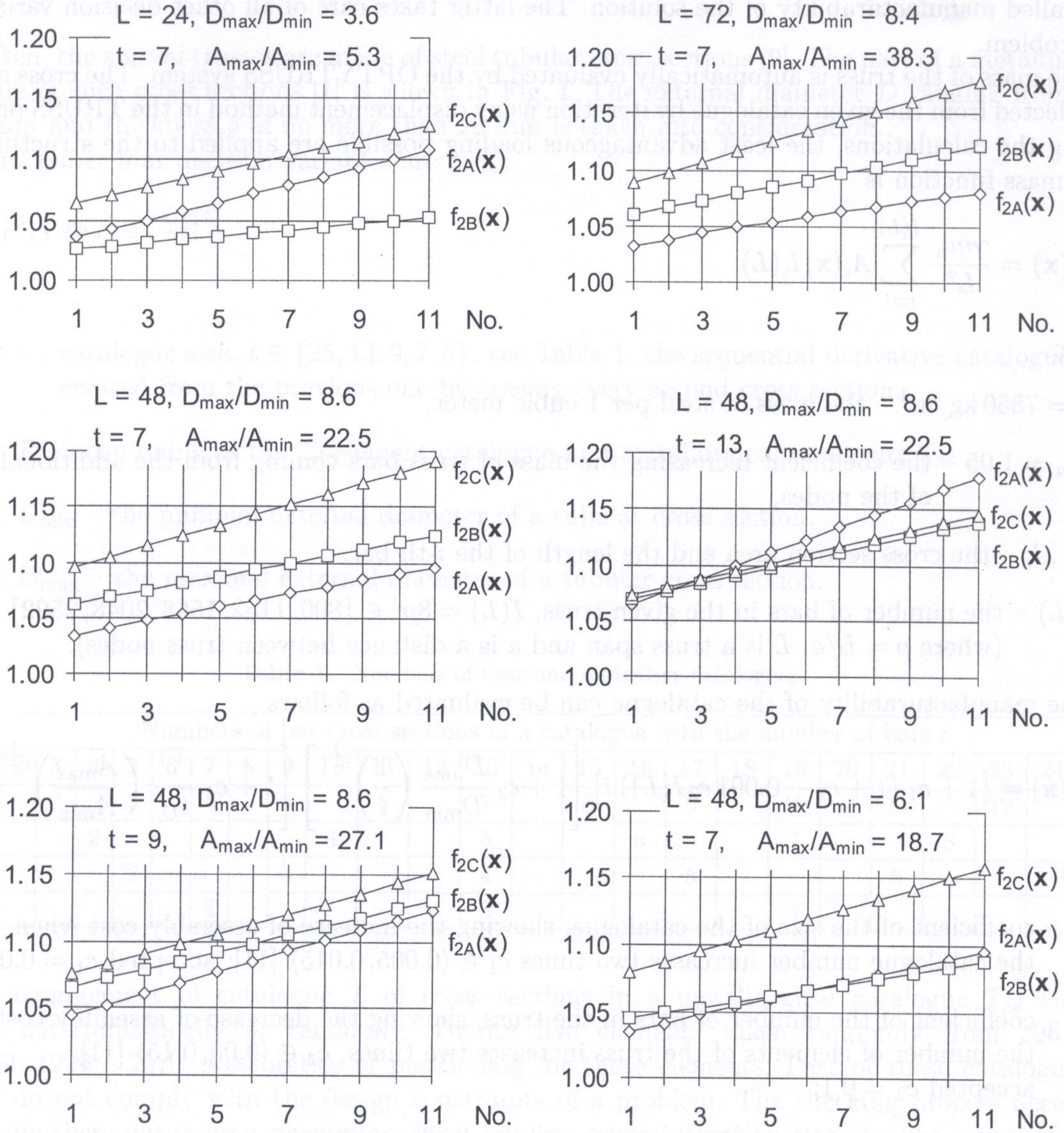
c_4 – coefficient of element marking costs $c_4 \in \langle 0.04, 0.08\rangle$; accepted $c_4 = 0.06$;

$A_{\min}$, $A_{\max}$ – minimal and maximal cross section areas of elements in the catalogue T;

t_D – the number of elements having different diameter D in the catalogue.

The so-called manufacturability function has been constructed to take into account all such local factors as: experience and backlog of orders of a contractor, access to a full metallurgical catalogue, an automation level of assembly, costs of elements per unit. The c_i coefficients should be such as to comply with all possible market situations. Their influence on the $f_2(\mathbf{x})$ function is shown in Fig. 1. In the optimization problem, the mean values of all the coefficients have been taken into account.

Function $f_2(\mathbf{x})$ describes manufacturability of particular solutions belonging to a domain of feasible solutions. It is based on taking into account mainly an influence of assumed design variables



No.	1	2	3	4	5	6	7	8	9	10	11
c_1	0.005	0.006	0.007	0.008	0.009	**0.010**	0.011	0.012	0.013	0.014	0.015
c_2	0.050	0.060	0.070	0.080	0.090	**0.100**	0.110	0.120	0.130	0.140	0.150
c_3	0.010	0.011	0.012	0.013	0.014	**0.015**	0.016	0.017	0.018	0.019	0.020
c_4	0.040	0.044	0.048	0.052	0.056	**0.060**	0.064	0.068	0.072	0.076	0.080

Fig. 1. The variation range of the manufacturability function $f_2(\mathbf{x})$ according to c_i coefficients

on manufacturability of solutions and assigning smaller values of the function $f_2(\mathbf{x})$ to the solutions having better manufacturability. Therefore, the function $f_2(\mathbf{x})$ should be minimized.

Manufacturability of solutions, under given vector of design variables $\mathbf{x}$, depends mainly on the angle α of inclination diagonal bars to planes of upper and lower surface layers (Fig. II), i.e., $\alpha = \arctan h\sqrt{2}/a$. The more the angle α tends to $\pi/4$, the smaller is the size of the joints and it is easier to manufacture them [12].

The criterion for the polyoptimal problem is the minimum of both objective functions:

$$\operatorname*{Min}_{\mathbf{x}\in\mathbf{X}} \mathbf{f}(\mathbf{x}) = \operatorname*{Min}_{\mathbf{x}\in\mathbf{X}} [f_1(\mathbf{x}),\, f_2(\mathbf{x})]^T. \tag{5}$$

All the constraints on feasible solutions are shown in Table 2. The design constraints (6a–6e) (according to Polish codes [13]), computational constraints (7a–7c), and technological constraints (8a–8k) have been taken into account. The design constraints take care of load carrying ability of tensioned (6a) and compressed (6b) bars, buckling of compressed bars (6c), cross sections selection in each iteration (6d), and limit serviceability state (6e). Computational constraints are used to control the TRUSS program. Different levels of termination criteria: $z_{\%} = 80\%$ and $z_m = 0.03\%$ give results with enough accuracy for construction design, including optimization. With that level of accuracy, the TRUSS program makes from four to ten design iterations, depending on the number of catalogue and number of loading conditions. Termination criteria $z_{\%} = 100\%$ and $z_m = 0\%$ should be applied to preferred solutions and to these used in practice. Technological constraints come from decision variable discretization and other variables depending of them. Constraints (8k) concern slenderness of bars under tension. The regular truss structures usually have in its middle sectors a group of bars having rather small tension force. With no constraints (8k) it would imply selecting for them too slender an element. The minimal cross section constraint takes care of that. Technological constraints (8b), (8c), (8e) and (8j) assure the set $\mathbf{X}$ to be discrete.

Table 2. Optimization constraints

Design constraints		Technological constraints	
$N_i^t \le A_i f_d\,, \quad i = 1,\ldots, I(L)$	(6a)	$2.4\,\text{m} \le a \le 4.0\,\text{m}$	(8a)
$N_i^c \le \phi_i A_i f_d$	(6b)	$10 \le (p = L/a) \le 18$	(8b)
$\lambda_i^c = \dfrac{\mu l_i}{i_i} \le 250$	(6c)	$L/26 \le h \le L/14,\ h/0.1 \in C$	(8c)
$[\mathbf{K}]_v\{\delta\}_{v\,c_l} = \{\mathbf{P}\}_{v\,c_l}$	(6d)	$44^\circ \le [\alpha = \arctan(\sqrt{2}h/a)] \le 46^\circ$	(8d)
$\delta_z \le \dfrac{L}{250} \in \langle 9.6, 28.8\,\text{cm}\rangle$	(6e)	$A_i \in T \quad T_M$	(8e)
		$2.9 \le g \le 20.0\,\text{mm}$	(8f)
Computational constraints		$31.8 \le D \le 508.0\,\text{mm}$	(8g)
$z_{\%} \in \langle 80, 100\%\rangle$	(7a)	$3 \le D_{\max}/D_{\min} \le 12$	(8h)
$z_m \in \langle 0.0, 0.03\%\rangle$	(7b)	$1 \le t \le 25$	(8i)
$c_l \in C_l = \{1, 9, 17\}$	(7c)	$t \in \{25, 13, 9, 7, 5\}$	(8j)
		$\lambda^t_{\max} = \dfrac{l_{\max}}{i_{\min}} = \dfrac{400\,\text{cm}}{1.03\,\text{cm}} = 388 < 500$	(8k)

where: N_i^t, N_i^c – tension (^{t}) and compression (^{c}) of the i-th element, ϕ_i – buckling parameter of the i-th element, λ_i^c, i_i – slenderness ratio and radius of inertia of the i-th element, $[\mathbf{K}]_v$ – stiffness matrix in the v-th iteration, $\{P\}_{v\,c_l}$ – load vector in the v-th iteration and c_l-th load combination, δ_z – limiting vertical displacement of a truss node, C_l – set of numbers of load combinations, $z_{\%}$ required rate of bars that do not change the cross-sections in consecutive iterations, z_m – permissible rate of variation of the mass of the truss in consecutive iterations, α – angle between the horizontal truss layer and its diagonals, C – the set of integers.

3. The polyoptimization object

The polyoptimization object is a tubular cross sections catalogue of bars for spatial truss cover of a hall. The dimensions of halls under consideration vary from $L \times L = 24 \times 24$ m to $L \times L = 72 \times 72$ m, see Fig. II. The trusses are supported at every other node of the upper layer. Horizontal bracing in walls is such that it prevents the horizontal displacement of the upper end of supporting column to be larger than 1/250 of the hall height. The spatial trusses with orthogonal meshes are to be made of steel grade R35 ($f_d = 210$ MPa).

The important properties of trusses with spans from 24 to 72 m are presented in Table 3. For the purpose of this paper, a model of the truss with bar-node type loaded only at nodes has been adopted. The loads consist of: dead load automatically modified after every iteration ($p_1 =$ 110–530 N/m^2, load factor – $\gamma_{f1} = 1.1$), roof covering load ($p_2 = 430$ N/m^2, $\gamma_{f2} = 1.171$), snow load ($p_3 = 560$ N/m^2, $\gamma_{f3} = 1.4$), technological loading ($p_4 = 500$ N/m^2, $\gamma_{f4} = 1.4$) and wind load ($p_5 = 450$ N/m^2, $\gamma_{f5} = 1.3$), according to applicable codes.

Table 3. Properties of the analyzed trusses

L	a	p	h	H	z	k	ls*
m	m	L/a	m	m	m	kN/m	
24.0	2.40	10	1.7	6.0	0.96	505	2
36.0	3.00	12	2.1	8.0	1.44	746	2
48.0	3.43	14	2.4	10.0	1.92	976	2
60.0	3.75	16	2.6	12.0	2.40	798	3
72.0	4.00	18	2.8	14.0	2.88	945	3

* ls – number of bracings along every wall of the hall

Every truss element must comply with the maximal loading as a result of any loading condition combinations (Table 4). Each loading has its own so called *loading factor* γ_{fi} and its own coincidence factor Ψ_i according to Polish Loading Codes [13]. The loading charts are shown in Fig. 2.

Wind load with its vector of loading coming from many different directions leads to many loading condition combinations shown in Fig. 2. In the preliminary analysis, to evaluate maximal cross section for a truss element, it is enough to take into account loading without any wind load. In the polyoptimal design, five loading condition combinations have been taken into account. The wind directions $+X, -X, +Y, -Y$ have been marked according to a global coordinate system. The final decision concerning preferred solutions should also take into account wind loading vectors coming at 45 degree to the hall wall. Table 4 and Fig. 2 show these directions as $+X+Y$, $+X-Y$, $-X+Y$, $-X-Y$.

4. The polyoptimal selection of cross sections catalogue

In the stage of the polyoptimal selection of cross sections, the specially designed program OPTYTRUSS for spatial truss optimization has been used. With that program it was possible to formulate and solve a polyoptimal problem of a real spatial truss having a couple of different spans. The construction and load parameters have been selected in such a way (Tables 1, 2, 3, 4) that for different spans L the results could be comparable. In the first stage of selection, the necessary maximal cross section area $A_{\max}(L)$ complying with all design constraints (6a, b) and suitable for the least advantageous loading condition possible has been found. Its diameter D and thickness g guarantee that the last element of the catalogue T_B is positioned on the lower boundary of thickness g for the metallurgical catalogue T_M. The technological constraints (8f–h, 8k) helped to establish the first element of the catalogue. For spans L = 24–48 m it seems to be the minimal element of the catalogue T_M, for L = 60 m the first element has cross-section parameters D = 42.4 and

Table 4. Loading factors γ_{fi} and their coincidence factors Ψ_i in analyzed loading condition combinations of spatial trusses

Truss analysis	No. of load combination c_l	p_1		p_2		p_3		p_4		p_5 $+X$		p_5 $-X$		p_5 $+Y$		p_5 $-Y$		p_5 $+X+Y$		p_5 $+X-Y$		p_5 $-X+Y$		p_5 $-X-Y$	
		γ_{f1}	Ψ_1	γ_{f2}	Ψ_2	γ_{f3}	Ψ_3	γ_{f4}	Ψ_4	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5	γ_{f5}	Ψ_5
preliminary	1	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8
poly-optimal	2	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	1.3	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8
	3	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	1.3	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8
	4	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	1.3	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8
	5	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	0.0	0.8	1.3	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8
	6	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	1.3	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0
	7	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	1.3	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0
	8	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	1.3	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0
	9	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	1.0	1.3	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0
final	10	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	1.3	0.8	0.0	0.8	0.0	0.8	0.0	0.8
	11	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	1.3	0.8	0.0	0.8	0.0	0.8
	12	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	1.3	0.8	0.0	0.8
	13	1.1	1.0	1.171	1.0	1.4	1.0	1.4	0.9	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	0.0	0.8	1.3	0.8
	14	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	1.3	1.0	0.0	1.0	0.0	1.0	0.0	1.0
	15	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	1.3	1.0	0.0	1.0	0.0	1.0
	16	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	1.3	1.0	0.0	1.0
	17	1.1	1.0	1.171	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	1.0	1.3	1.0

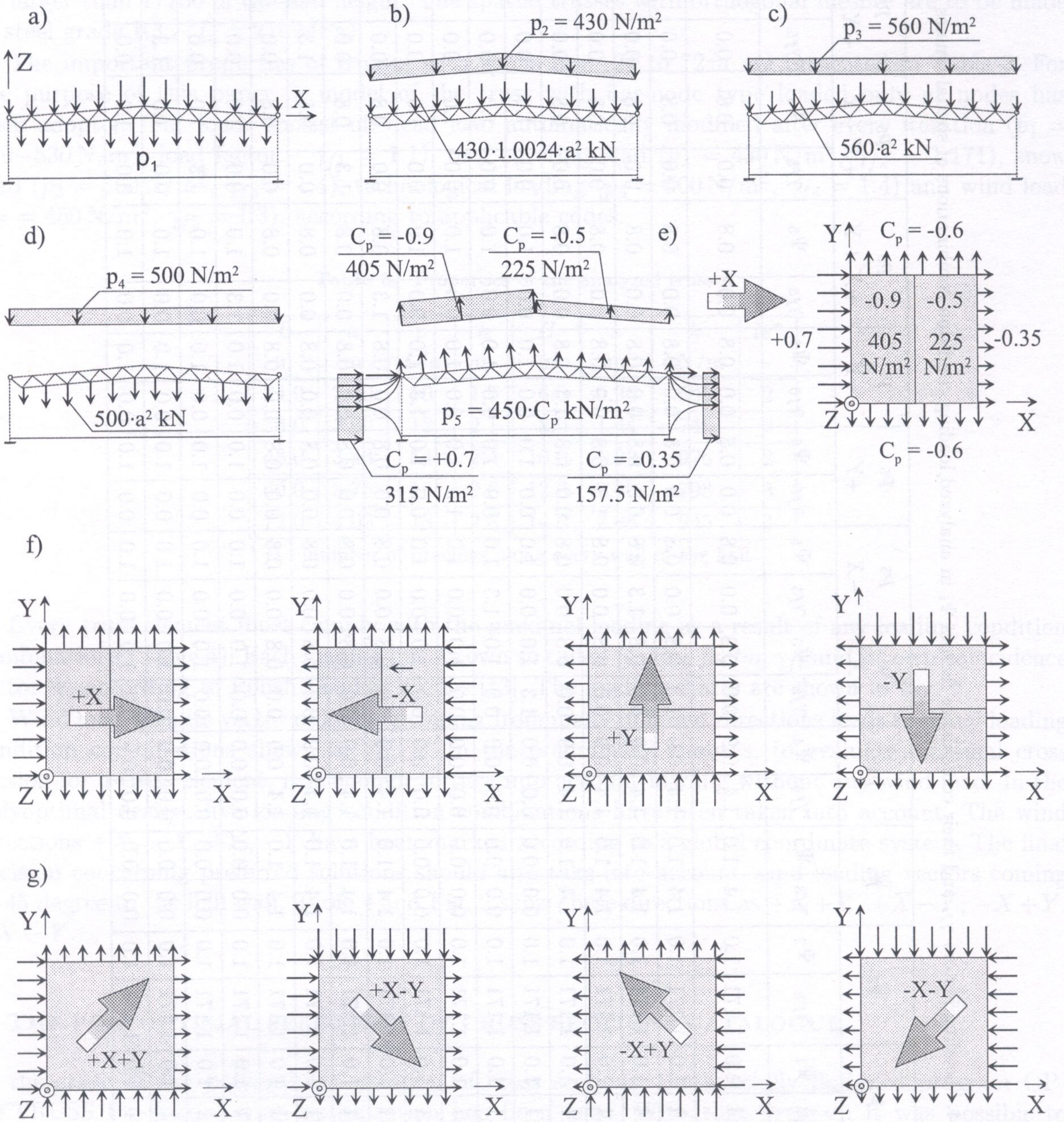


Fig. 2. Load schemes of a spatial truss: a) dead load, b) roof covering load, c) snow load, d) technological loading, e) wind load, f) wind load with its vector vertical to the wall, g) wind load with its vector at 45 degree to the wall surface

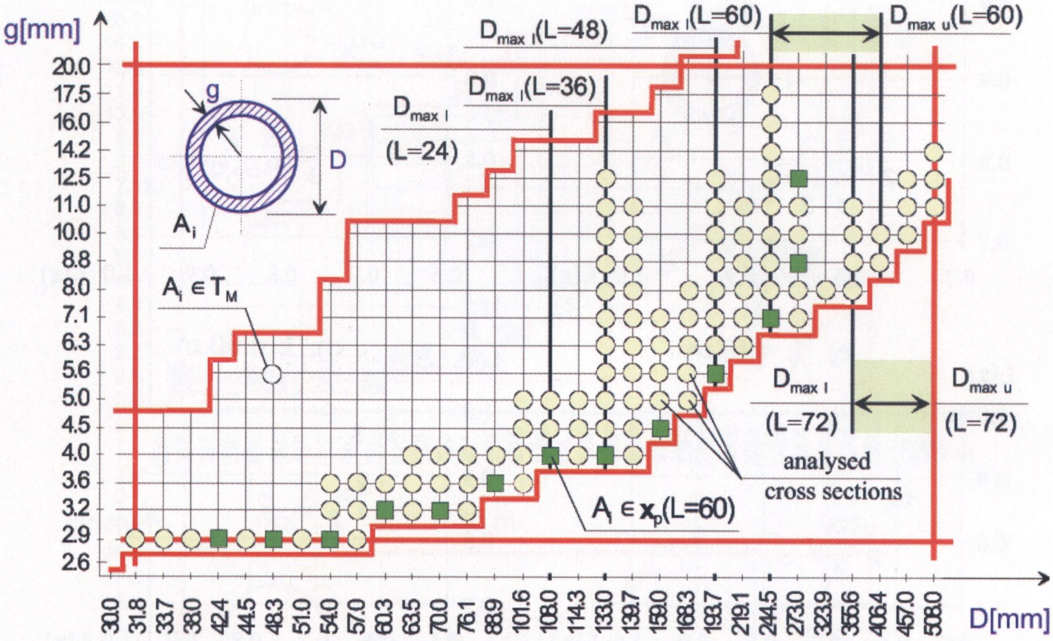


Fig. I. The cross sections being analysed and the preferred catalogue for $L = 60\,\mathrm{m}$

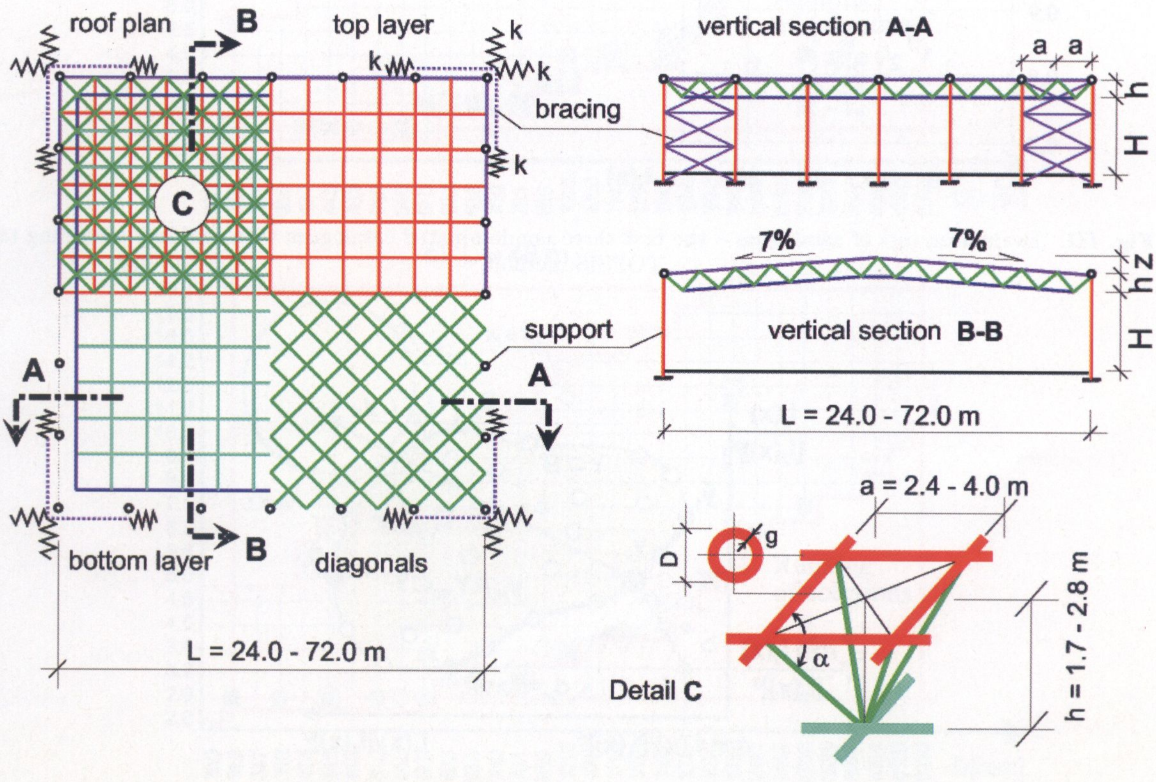


Fig. II. Layout of steel halls and spatial trusses used for roof coverings

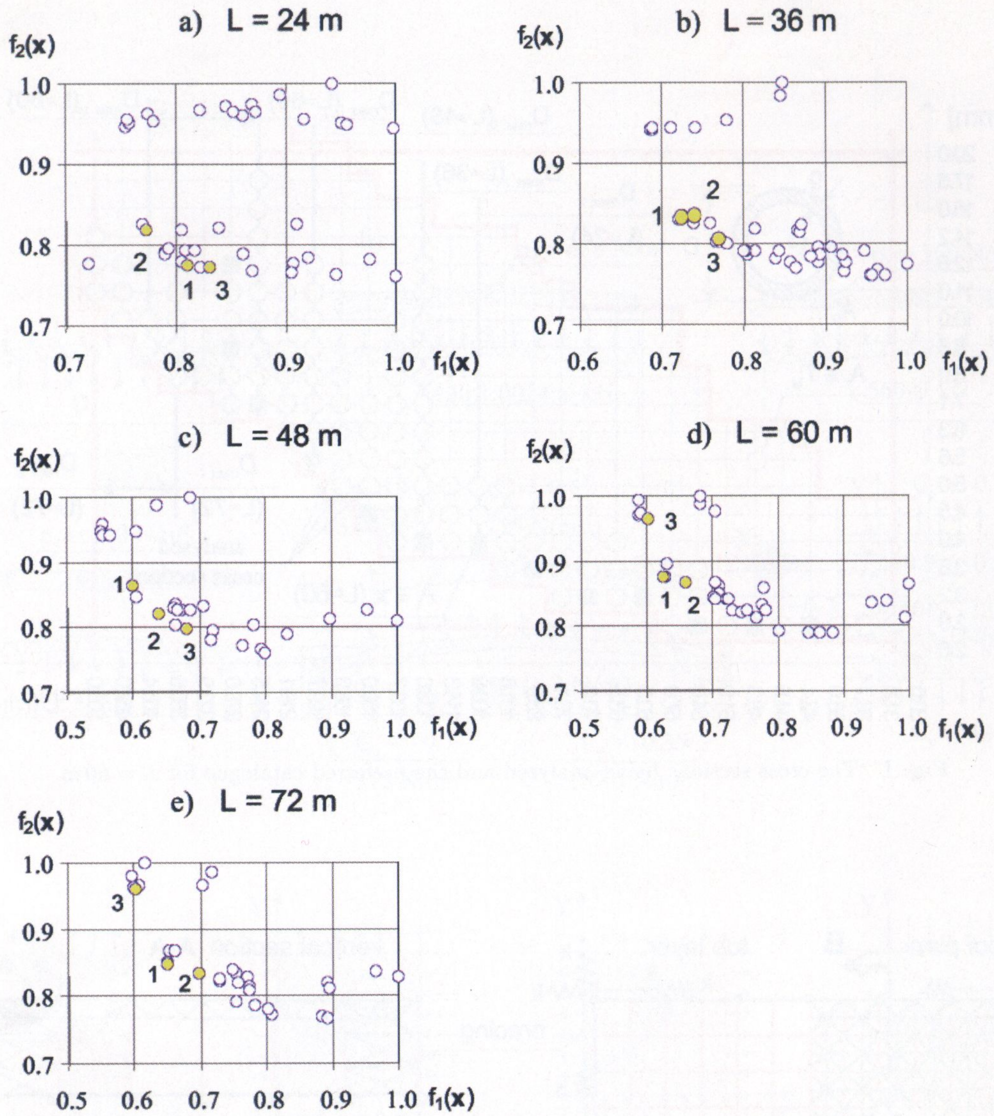


Fig. III. Evaluation sets of catalogues – the first three nondominated catalogues with ranking, according to TOPSIS method

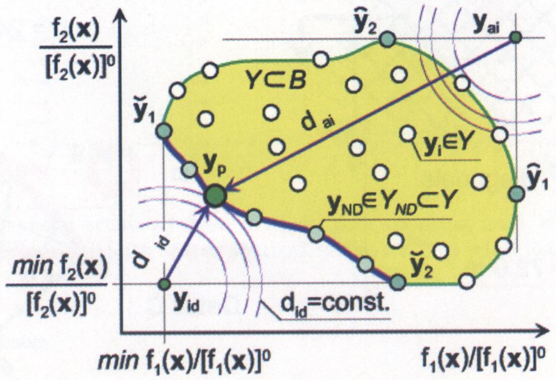


Fig. IV. Graphical illustration of the TOPSIS method

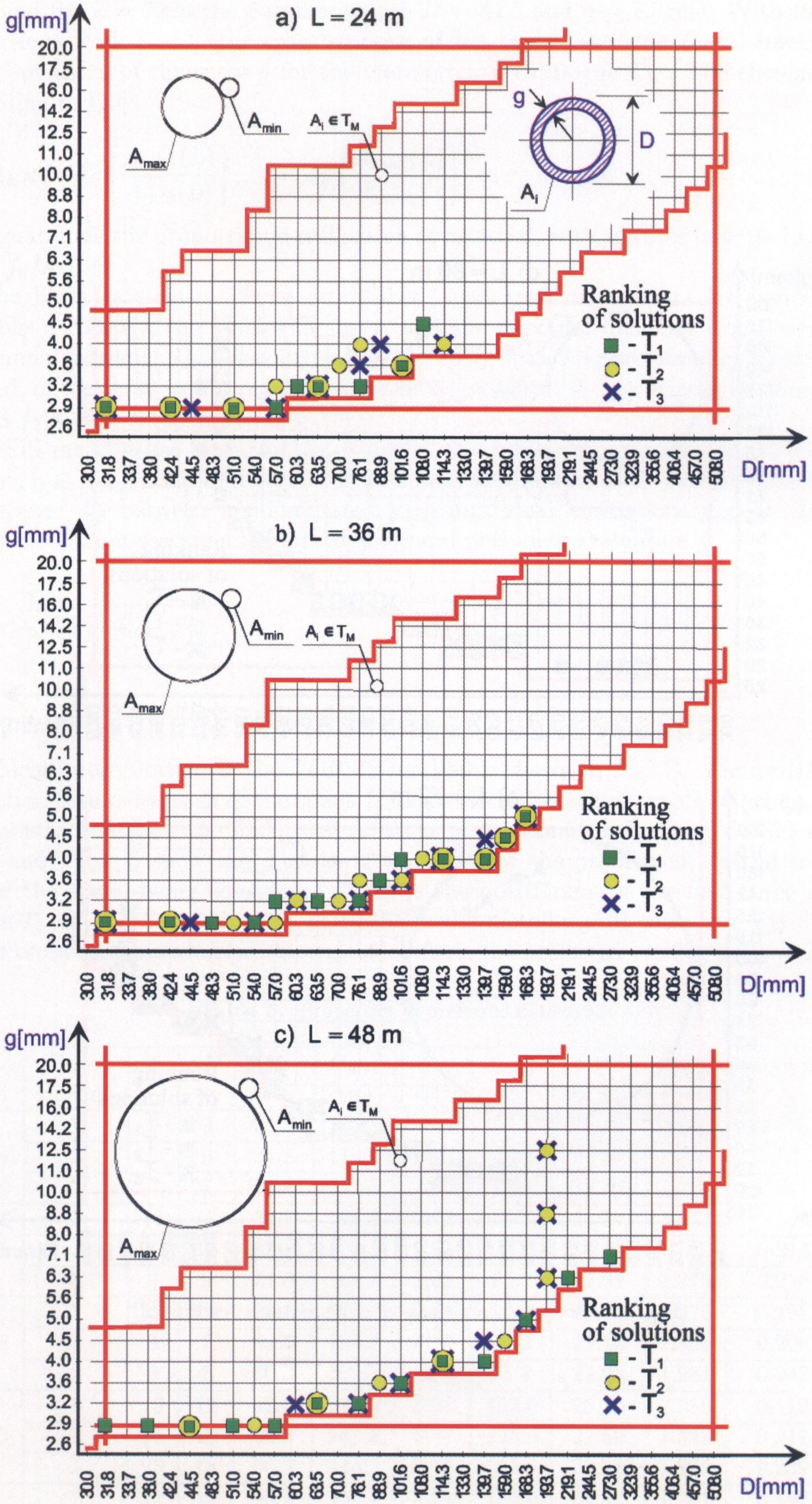


Fig. V. Preferred catalogues of cross sections *(continued in the next page)*

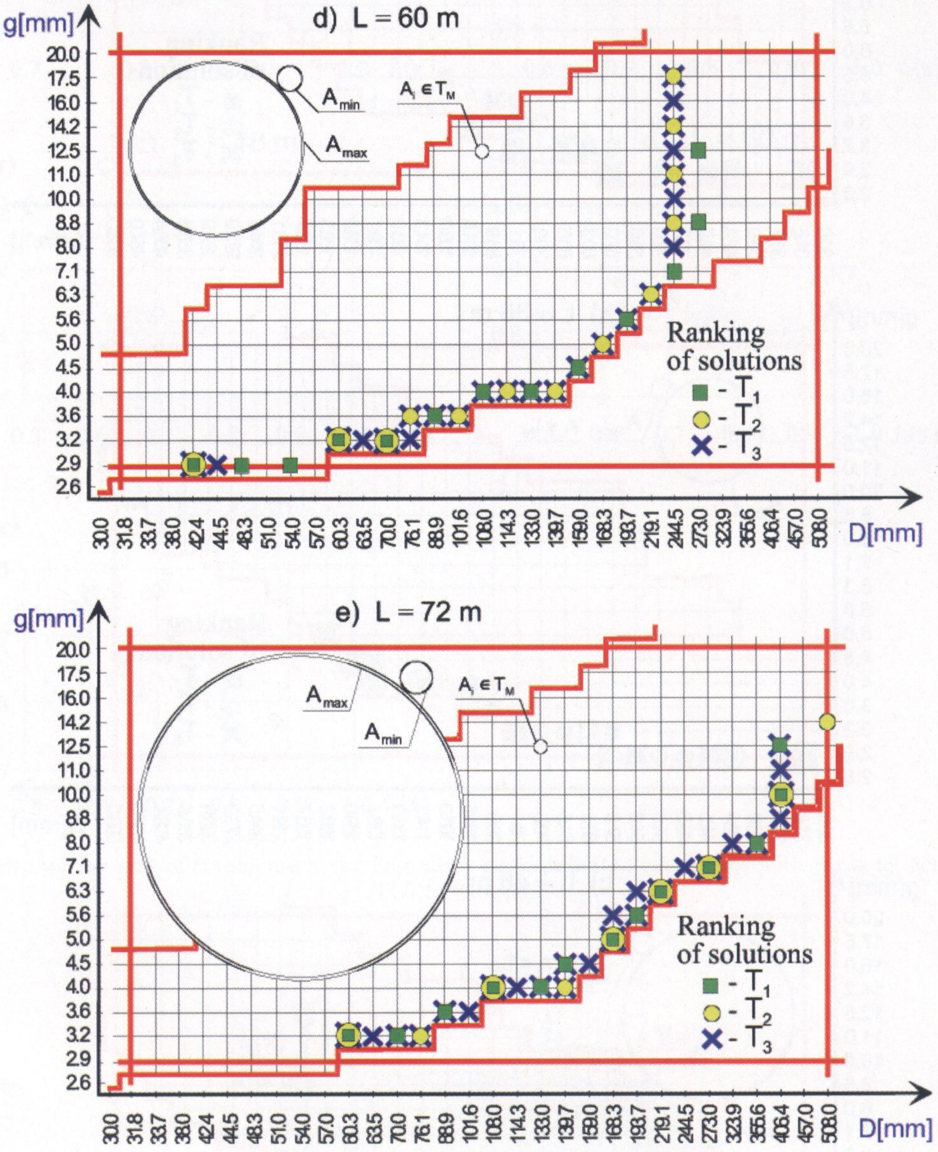


Fig. V. Preferred catalogues of cross sections *(continued)*

$g = 2.9$ mm and for $L = 72$ m the parameters are $D = 44.5$ and $g = 2.9$ mm. With the experience from the previous work [7–10], the cross sections of the basic catalogue $T_B(L)$ have been chosen at the lower boundary of thickness g for the metallurgical catalogue T_M. The elements have been chosen according to [10]:

$$A_{k+1} = A_k\,\kappa = A_k \left[\frac{A_{\max}(L)}{A_{\min}(L)}\right]^{\frac{1}{t-1}} = A_k \left[\frac{A_{\max}(L)}{A_{\min}(L)}\right]^{\frac{1}{24}}. \tag{9}$$

That would guarantee the proportional utilization of material, with stresses in truss bars being keep in the range $f_d/\kappa - f_d$.

Having the basic catalogues $T_B(L)$ established, next some constraints are varied (treated as design variables). Namely, the catalogue size t, and the maximal diameter $D_{\max}$ were decreased, while the minimal diameter $D_{\min}$ was increased. For every span L approximately 50 catalogues have been analyzed, of which 30 to 40 complied with the constraints (6)–(8). Figure III shows the results for $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ for different spans L.

The TOPSIS method has been used to establish the preferred catalogue. Every nondominated evaluations $\mathbf{y}_{ND} \in \mathbf{Y}_{ND}$ have been vectored and then normalized. The TOPSIS function $f_t(\mathbf{y}_{ND})$ measures distance d_{id} between nondominated $\mathbf{y}_{ND}$ and ideal evaluations $\mathbf{y}_{id}$. It also takes into account distance d_{ai} between nondominated $\mathbf{y}_{ND}$ and anti-ideal evaluation $\mathbf{y}_{ai}$,

$$f_t(\mathbf{y}_p) = \underset{\mathbf{y}^i_{ND}\in\mathbf{Y}_{ND}}{\mathrm{Min}} \frac{d^i_{id}}{d^i_{id} + d^i_{ai}}, \tag{10a}$$

where:

$$d^i_{id} = \left|\mathbf{y}^i_{ND} - \mathbf{y}_{id}\right|_{||2||}, \qquad d^i_{ai} = \left|\mathbf{y}^i_{ND} - \mathbf{y}_{ai}\right|_{||2||}. \tag{10b}$$

The graphical interpretation of the TOPSIS method is shown in Fig. IV. Figure III shows polyoptimal selection results for each of the spans L. Light circles show preferable solutions. Dark circles with numbers attached show the first three rankings of solutions got by the TOPSIS method.

Figure V and Table 5 show the domain of solutions for the problem of spatial truss polyoptimization. For the spans being considered it shows the positioning of the first three catalogues of cross sections T_1, T_2, T_3 (according to TOPSIS) in the metallurgical catalogue T_M. The minimal and maximal cross sections are also shown, scaled from the preferred catalogue $T_p = T_1$.

Table 5. Properties of preferred catalogues

L m	Topsis ranking	t	tD	$D_{\min}$ mm	$D_{\max}$ mm	$A_{\min}$ cm^2	$A_{\max}$ cm^2	$f_1(\mathbf{x})$ kg/m^2	$f_2(\mathbf{x})$ –	$f_t(\mathbf{y}_{ND})$ –
24	1	9	9	31.8	108.0	2.63	14.6	11.08	1.239	0.127
	2	9	9	31.8	114.3	2.63	17.2	11.07	1.244	0.131
	3	7	7	31.8	114.3	2.63	17.2	11.30	1.224	0.151
36	1	13	13	31.8	168.3	2.63	25.7	14.18	1.318	0.194
	2	13	13	31.8	168.3	2.63	28.6	14.45	1.319	0.214
	3	9	9	31.8	168.3	2.63	25.7	15.08	1.273	0.234
48	1	13	13	31.8	273.0	2.63	59.3	19.75	1.376	0.191
	2	9	7	44.5	193.7	3.79	71.2	21.12	1.305	0.206
	3	9	7	60.3	193.7	5.74	71.2	21.93	1.281	0.242
60	1	13	12	42.4	273.0	3.94	102.0	26.20	1.356	0.172
	2	13	10	60.3	244.5	5.74	125.0	27.59	1.343	0.211
	3	25	18	60.3	244.5	5.74	125.0	25.21	1.495	0.239
72	1	13	12	60.3	406.4	5.74	155.0	34.47	1.361	0.196
	2	9	9	60.3	508.0	5.74	220.0	36.96	1.338	0.260
	3	25	19	60.3	406.4	5.74	155.0	31.84	1.537	0.297

5. Conclusions

The cross sections in the preferred catalogues occur at the lower boundary of wall thickness of tubular cross sections of the metallurgical catalogue (Fig. I). Application of the function (9) enables uniform distribution of construction material and minimizes the construction weight.

The spatial truss seems to be the more useful the smaller the ratio $D_{\max}/D_{\min}$. This condition is shown at the catalogues in Fig. I and Fig. Vd,e, where two or three last cross-sections have the same diameter $D_{\max} \equiv D_{\max l}$. Then the preferred number of different elements in the catalogue equals 9 to 13. The spatial trusses of span $L = 24$–$36\,\mathrm{m}$ should be assembled from 7–9 different elements and of span $L = 48$–$72\,\mathrm{m}$ from 9–13 different elements.

The proper selection of the cross section catalogue effects the whole design of spatial trusses.

Acknowledgment

The authors gratefully acknowledge the partial support of the Polish Committee for Scientific Research (Grant No. 7T07A05212).

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