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Manufacturing tolerances of truss members' lengths in minimum weight design

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In most cases a safety of optimal construction may be limited by the violation of stress, buckling or displacement constraints. An unexpected exceed of these constraints may be caused by manufacturing tolerances of structural elements (differences between assumed and obtained dimensions). This requires an incorporation of tolerance problem in optimum design. One may deal with two different tolerances – the first case is when it's related to the members' cross-section variations, whereas the second notion represents the variation of elements' lengths. Considering operation conditions and manufacturing techniques the second case of tolerance seems to be more important. This approach states the problem of minimum weight design of a structure with initial distortions. A standard solution algorithm with the Kuhn–Tucker theorem was used with the adjoint variable method. Necessary optimality conditions have the form of equations and inequalities. The equality constraints were put forward for the average values of design variables l, while tolerances t_j were introduced into inequality equations i.e. the limit values of stresses and displacements were diminished by the positive products of appropriate sensitivities and tolerances. The method was next illustrated by an example of a ten bar bench-mark problem – a typical one for testing algorithms in structural optimization. The idea presented in this paper may be used not only for truss structures but it can be easily extended to other kinds of structures like frames, composites etc.

Keywords: structural optimization, manufacturing tolerances, nonlinear optimization

1. Introduction

The development of the structural optimization and applying advanced computational techniques leads to a near-optimal solution for the stated problem. Having found the theoretical solution one or several constraints come to their limit values. In most real cases the safety of structure may be limited by the violation of stress, buckling or displacement constraints. An unexpected exceed of these constraints may be caused, among other things like material imperfections etc., by the manufacturing tolerances of structural elements (differences between assumed and obtained dimensions). This requires an incorporation of tolerance analysis in an optimum design problem. The importance of tolerance influence on the optimum design has drawn attention in a number of publications. In [11], a review of several works devoted to this subject is presented. Next, in this paper the problem of minimizing the margin of safety is considered. This is done by introducing the notion of "tolerance box" and is illustrated with a minimum weight design of a composite structure.

Two main groups of problems in tolerance analysis seem to appear: the first one, when the tolerances are *prior* given and a designer seeks for an optimum design under imposed constraints;

and the second one – while looking for a minimum cost of a structure, keeping the tolerances within a specified region [14]. The present paper deals with the first group of problems.

For truss structures, the mentioned above problem of minimum weight design was usually considered allowing for the elements' cross section tolerance [7]. Dealing with the operation conditions and manufacturing techniques the elements' lengths tolerance seems to be more important, but it hasn't been examined in detail yet. Some approaches to that subject were presented in papers [2, 3, 10] as a distortion problem.

At present the elements' manufacturing tolerances are minimized by the proper assembly technology – i.e. the lengthened rod and a nodal plate will overlap more, and will be welded in such position. The mentioned distortions may arise while assembling final structure from distorted subsystems, since, in general, most feasible structures are statically indeterminate. Therefore both the influence of elements' lengths tolerances (initial distortions) in the structure on its minimum weight and the reciprocal interactions between loads and initial structure distortions as well, need detailed examination.

2. Inequality constraints allowing for manufacturing tolerances

Considering a set of elements with the lengths' tolerance equal to zero (initial design) and assigning to each member an appropriate value l, one gets a series of l_j values. Constraints imposed on displacements and stresses take the known form

$$u_i^0 - u_i \geq 0; \quad \sigma_j^0 - \sigma_j \geq 0 \Leftrightarrow \sigma_j^0 - E\sum_{i=1}^{i=i_0} B_{ji} u_i \geq 0 \qquad \text{for} \quad i = 1 \ldots i_0\,, \quad j = 1 \ldots j_0\,, \tag{1}$$

where u_i and u_i^0 denote respectively the elements of nodal displacement vector $\mathbf{u}$ and limit values of possible displacements vector $\mathbf{u}^0$, σ and σ^0 are elements of stress vector $\boldsymbol{\sigma}$ and admissible stress vector $\boldsymbol{\sigma}^0$. Matrix $\mathbf{B}$ is the geometric matrix (component of stiffness matrix $\mathbf{K}$ applied in FEM) and E denotes the elasticity modulus (all components are manufactured from the same material).

Assume, in general, that the length of an initial structural member l_j varies from $l_j - t_j$ up to $l_j + t_j$ where t_j represents manufacturing tolerance. These variations will cause variations of nodal displacements and elements' stresses. Focusing attention on the i^{th} node we may find that its displacement, according to j^{th} member tolerance, is as follows,

$$u_i := u_i + s_{ij}^u t_j\,, \tag{2}$$

where the right sided u_i denotes a displacement under the condition of zero tolerance and s_{ij}^u denotes the sensitivity of i^{th} displacement to j^{th} elements length. Extending the considerations to all $j = 1, \ldots, j_0$ elements we find the total displacement as a function

$$u_i := u_i + \sum_{j=1}^{j_0} s_{ij}^u t_j\,. \tag{3}$$

The similar considerations may be derived for stresses:

$$\sigma_i := \sigma_i + \sum_{j=1}^{j_0} s_{ij}^{\sigma} t_j\,. \tag{4}$$

In a general case the product of t_j and s_{ij}^u or s_{ij}^u can take positive or negative values. This means it can have "positive" or "negative" influence on present displacements or stresses. As the total number of all possible combinations is difficult to evaluate and moreover their exact interaction with the values of state variables is hard to estimate – it's justified to introduce absolute value of the product into inequality constraints.

The mentioned above reasons cause the necessity of constraints' modification. In general, the constraints imposed on stresses and displacements are as follows,

$$\mathbf{u}^0 - \mathbf{u} \geq \mathbf{0}\,, \qquad \boldsymbol{\sigma}^0 - \boldsymbol{\sigma} \geq \mathbf{0}\,,$$

and considering tolerances they will get the new form

$$\mathbf{u}^0 - [\mathbf{u} + |\mathbf{st}|] \geq \mathbf{0}\,, \qquad \boldsymbol{\sigma}^0 - [\boldsymbol{\sigma} + |\mathbf{st}|] \geq \mathbf{0}\,.$$

3. Problem statement

Consider a truss structure with i_0 joints and j_0 elements. The truss members are manufactured from the same material each, with a given elasticity modulus E, and with known bounds on lengths' tolerances t_j. For quantitative study of the influence of tolerances on optimum design we assume the linear dependence of t_j with respect to l_j – i.e. $t_j := \mu \cdot l_j$. Without restrain of generality of our considerations, we can assume constant μ for every truss member j.

The task of our optimization problem is to find a minimum of a structure volume

$$f = \mathbf{A}^T\,\mathbf{l} \tag{5}$$

with imposed equality and inequality constraints – $\mathbf{h}$ and $\mathbf{g}$, respectively. Above, $\mathbf{A} = [A_1\,, A_2\,, \ldots\,, A_{j_0}]^T$ stands for the cross-sectional dimensions vector and $\mathbf{l} = [l_1\,, l_2\,, \ldots\,, l_{j_0}]^T$ stands for the elements' lengths.

Previously mentioned sensitivity matrices $\mathbf{s}^u$ and $\mathbf{s}^\sigma$, where subscript (i) or (j) denotes the i^{th} (j^{th}) constraint, are defined for initial design Eq. (1) as

$$\mathbf{s}^u_{(i)} = \frac{\mathrm{d}\mathbf{g}_{(i)}}{\mathrm{d}\mathbf{l}} = \frac{\partial \mathbf{g}_{(i)}}{\partial \mathbf{l}} + \frac{\partial \mathbf{g}_{(i)}}{\partial \mathbf{u}}\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{l}} \qquad \text{for} \quad i = 1, \ldots, i_0\,, \tag{6}$$

$$\mathbf{s}^\sigma_{(j)} = \frac{\mathrm{d}\mathbf{g}_{(j)}}{\mathrm{d}\mathbf{l}} = \frac{\partial \mathbf{g}_{(j)}}{\partial \mathbf{l}} + \frac{\partial \mathbf{g}_{(j)}}{\partial \mathbf{u}}\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{l}} \qquad \text{for} \quad j = i_0 + 1, \ldots, i_0 + j_0\,. \tag{7}$$

The above equations were solved according to the adjoint variable method [9].

Introducing state equation

$$\mathbf{K}\,(\mathbf{A}, \mathbf{l}) \cdot \mathbf{u} - \mathbf{P} = \mathbf{0} \tag{8}$$

one gets its derivative

$$\frac{\mathrm{d}\mathbf{K}(\mathbf{A}, \mathbf{l})}{\mathrm{d}\mathbf{l}}\,\mathbf{u} + \mathbf{K}(\mathbf{A}, \mathbf{l})\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{l}} - \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}\mathbf{l}} = \mathbf{0} \quad \Rightarrow \quad \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}\mathbf{l}} = -\mathbf{K}^{-1}(\mathbf{A}, \mathbf{l})\,\frac{\mathrm{d}\mathbf{K}(\mathbf{A}, \mathbf{l})}{\mathrm{d}\mathbf{l}}\,\mathbf{u}. \tag{9}$$

The above put into Eq. (6) gives

$$\mathbf{s}^u_{(i)} = \frac{\partial \mathbf{g}_{(i)}}{\partial \mathbf{u}}\,\mathbf{K}^{-1}(\mathbf{A}, \mathbf{l})\,\frac{\mathrm{d}\mathbf{K}(\mathbf{A}, \mathbf{l})}{\mathrm{d}\mathbf{l}}\,\mathbf{u}\,. \tag{10}$$

Define an *adjoint variable* as

$$\boldsymbol{\Phi}_{(k)} = \left[\frac{\partial \mathbf{g}_{(k)}}{\partial \mathbf{u}}\,\mathbf{K}^{-1}(\mathbf{A}, \mathbf{l})\right]^T = \mathbf{K}^{-1}(\mathbf{A}, \mathbf{l})\,\frac{\mathrm{d}\mathbf{g}^T_{(k)}}{\mathrm{d}\mathbf{u}}\,. \tag{11}$$

As the result we obtain the final expression for the sensitivity

$$\mathbf{s}^u_{(i)} = -\boldsymbol{\Phi}^T_{(i)}\,\frac{\mathrm{d}\mathbf{K}(\mathbf{A}, \mathbf{l})}{\mathrm{d}\mathbf{l}}\,\mathbf{u}\,. \tag{12}$$

Finally, we get a set of equations

$$\mathbf{s}_{(k)} + \mathbf{\Phi}^T_{(k)} \left(\frac{\partial \mathbf{K}}{\partial \mathbf{l}} \mathbf{u} \right) = 0, \tag{13}$$

$$\mathbf{K}(\mathbf{A},\mathbf{l})\, \mathbf{\Phi}_{(k)} - \frac{\mathrm{d}\mathbf{g}^T_{(k)}}{\mathrm{d}\mathbf{u}} = 0, \tag{14}$$

where: $\mathbf{s}_{(k)}$ denotes the general sensitivity vector of the k^{th} constraint with respect to all design variables, $\mathbf{\Phi}_{(k)}$ stands for the *adjoint variable vector* associated with the k^{th} sensitivity vector and finally $\mathbf{g}_{(k)}$ means the constraint (inequality) imposed on the k^{th} state variable.

Inequality constraints imposed on displacements, stresses (including tolerances) and cross-sections are as follows,

$$u_i^0 - \left[u_i + \sum_{j=1}^{j_0} \left| s^u_{ij} t_j \right| \right] \geq 0, \qquad i = 1, 2 \ldots, i_0, \tag{15}$$

$$\sigma_k^0 - \left[\sigma_k + \sum_{j=1}^{j_0} \left| s^\sigma_{kj} t_j \right| \right] \geq 0, \qquad k = 1, 2 \ldots, j_0, \tag{16}$$

$$\mathbf{A}_j - \mathbf{A}_j^{\min} \geq 0 \qquad \text{for} \quad j = 1 \ldots j_0. \tag{17}$$

4. Kuhn–Tucker necessary conditions

The stated problem belongs to the class of nonlinear programming methods described in many monographs e.g. [12]. It can be solved by one of the standard algorithms form the library of nonlinear programming methods, i.e. SLP or NLPQL. The authors decided to develop and improve their own previously worked out algorithm, based on Kuhn–Tucker necessary conditions [4, 5, 6]. The Kuhn–Tucker theorem [13] defining the necessary conditions for an optimum solution gives Lagrange'an in a form as below,

$$L = -\mathbf{A}^T \mathbf{l} + \boldsymbol{\lambda}^{e^T} (\mathbf{K}\mathbf{u} - \mathbf{P}) + \boldsymbol{\lambda}^{t^T} \left(\mathbf{s}_{(k)} + \mathbf{\Phi}^T_{(k)} \left(\frac{\partial \mathbf{K}}{\partial \mathbf{l}} \mathbf{u} \right) \right) + \boldsymbol{\lambda}^{a^T} \left(\frac{\partial \mathbf{g}^T_{(k)}}{\partial \mathbf{u}} - \mathbf{K}\mathbf{\Phi}_{(k)} \right)$$

$$+ \boldsymbol{\lambda}^{d^T} (\mathbf{u}^0 - \mathbf{u}) + \boldsymbol{\lambda}^{s^T} (\boldsymbol{\sigma}^0 - \mathbf{E}\mathbf{B}\mathbf{u}) + \boldsymbol{\lambda}^{c^T} (\mathbf{A} - \mathbf{A}^{\min}), \tag{18}$$

and moreover a system of following equations and inequalities,

$$\frac{\partial L}{\partial \boldsymbol{\lambda}^e} = \mathbf{K}(\mathbf{A},\mathbf{l}) \cdot \mathbf{u} - \mathbf{P} = \mathbf{0}, \tag{19}$$

$$\frac{\partial L}{\partial \boldsymbol{\lambda}^a} = \frac{\partial \mathbf{g}^T_{(k)}}{\partial \mathbf{u}} - \mathbf{K}(\mathbf{A},\mathbf{l})\, \mathbf{\Phi}_{(k)} = \mathbf{0}, \tag{20}$$

$$\frac{\partial L}{\partial \mathbf{u}} = \mathbf{K}(\mathbf{A},\mathbf{l})\, \boldsymbol{\lambda}^e - \boldsymbol{\lambda}^{d^T} - \boldsymbol{\lambda}^{s^T}\, \mathbf{E}\mathbf{B} - \boldsymbol{\lambda}^{t^T} \mathbf{\Phi}^T_{(\kappa)} \frac{\partial \mathbf{K}}{\partial \mathbf{l}} = \mathbf{0}, \tag{21}$$

$$\frac{\partial L}{\partial \Phi} = \boldsymbol{\lambda}^t \frac{\partial \mathbf{K}(\mathbf{A},\mathbf{l})}{\partial \mathbf{l}} \mathbf{u} - \boldsymbol{\lambda}^{a^T}\, \mathbf{K}(\mathbf{A},\mathbf{l}) = \mathbf{0}, \tag{22}$$

$$\frac{\partial L}{\partial \mathbf{s}} = \boldsymbol{\lambda}^{t^T}_{(k)} \pm \mathbf{s}_{(k)} \mu \boldsymbol{\lambda}^d \mathbf{A} = \mathbf{0}, \tag{23}$$

$$\frac{\partial L}{\partial \mathbf{A}} = -\mathbf{l} + \boldsymbol{\lambda}^{e^T} \frac{\partial \mathbf{K}}{\partial \mathbf{A}} \mathbf{u} + \sum_{k=1}^{i_0+j_0} \boldsymbol{\lambda}^{t^T}_{(k)} \frac{\partial \mathbf{K}}{\partial \mathbf{A}} \mathbf{\Phi}_{(k)} - \mu \boldsymbol{\lambda}^{d^T} \mathbf{s}^u_{(k)} - \mu \boldsymbol{\lambda}^{s^T} \mathbf{s}^\sigma_{(k)} + \boldsymbol{\lambda}^{c^T} = \mathbf{0}, \tag{24}$$

$$\boldsymbol{\lambda}^{d^T} \cdot \left[\mathbf{u}^0 - \left(\mathbf{u} + \sum_{j=1}^{jo} \left| \mathbf{s}^u_{(j)} \mu l_j \right| \right) \right] = 0 , \tag{25}$$

$$\boldsymbol{\lambda}^{s^T} \cdot \left[\boldsymbol{\sigma}^0 - \left(\boldsymbol{\sigma} + \sum_{j=1}^{jo} \left| \mathbf{s}^\sigma_{(j)} \mu l_j \right| \right) \right] = 0 , \tag{26}$$

$$\boldsymbol{\lambda}^{c^T} \cdot (\mathbf{A} - \mathbf{A}^{\min}) = 0 , \tag{27}$$

$$\boldsymbol{\lambda}^{d} , \boldsymbol{\lambda}^{s} , \boldsymbol{\lambda}^{c} \geq 0 , \tag{28}$$

where $\boldsymbol{\lambda}^e$, $\boldsymbol{\lambda}^a$, $\boldsymbol{\lambda}^t$, $\boldsymbol{\lambda}^d$, $\boldsymbol{\lambda}^s$ and $\boldsymbol{\lambda}^c$ are respectively vectors or matrices of Lagrange multipliers associated with equations of equilibrium, adjoint equation, sensitivity relation and constraints imposed on displacements, stresses and sizes.

As reported by many authors [1, 6] the main difficulty in solving the derived set of equations is related to the evaluation of the Lagrange multiplier associated with the active constraint (the all remaining are zero). To do this an extra relation was derived. First multiply (21) by $\mathbf{u}$ and (22) by $\boldsymbol{\Phi}$ and (24) by $\mathbf{A}$. Then consider the linear combination: subtract first two relations from the last one. One gets the following equation,

$$f = \boldsymbol{\lambda}^{d^T} \left(\mathbf{u} + \mu \left| \mathbf{s}_{(k)} \right| \mathbf{A} \right) + \boldsymbol{\lambda}^{s^T} \left(\boldsymbol{\sigma} + \mu \left| \mathbf{s}_{(j)} \right| \mathbf{A} \right) + \boldsymbol{\lambda}^{c^T} \mathbf{A}^{\min} . \tag{29}$$

Bearing in mind that Lagrange multipliers differ from zero only for active constraints, the above equation may be transformed to the following form,

$$f = \boldsymbol{\lambda}^{d^T} \mathbf{u}^0 + \boldsymbol{\lambda}^{s^T} \boldsymbol{\sigma}^0 + \boldsymbol{\lambda}^{c^T} \mathbf{A}^{\min} . \tag{30}$$

This equation will be used later in the proposed solution algorithm.

5. Solution algorithm

A solution algorithm is based on the interdependent actions of professional FEM module (analyzer) and optimization procedures arising from the Kuhn–Tucker theory (optimizer) [4, 5, 6].

During iterative procedures, all active constraint functions came down to their limit values. Applying scaling (step 5) at each iteration step the most violated constraint is brought to its limit value. So, at each step, one has a single active constraint problem. If so, then the Lagrange multiplier associated with this constraint may be derived from an additional relation (30). The remaining multipliers are equal to 0. Calculations in the step 7 are also carried out by FEM software.

Below, the grey colour denotes actions performed by optimization procedures and the white one means professional FEM module calculations:

Step 1 Take $n := 0$ where n denotes iteration counter.

Step 2 Assume an arbitrary vector $\mathbf{A}(0)$ of design variables A_j.

Step 3 Solve equations for $\mathbf{u}(0)$, $\boldsymbol{\sigma}(0)$ and $\mathbf{s}(0)$.

Step 4 Find

$$r(n) = \max \left\{ \max \left(\frac{u_i + \sum_{j=1}^{jo} s^u_{ij} t_j}{u^0_i} \right) ; \max \left(\frac{\sigma_k + \sum_{j=1}^{jo} s^\sigma_{kj} t_j}{\sigma^0_i} \right) ; \max \left(\frac{A^{\min}}{A_j} \right) \right\} .$$

Step 5 Scale design variables A_j and state variables u_j and σ_j with $r(n)$:

$$A_j(n) := A_j(n)\, r(n)\,; \qquad u_i(n) := u_i(n)/r(n)\,; \qquad \sigma_j(n) := \sigma_j(n)/r(n)\,.$$

Step 6 Find the Lagrange multiplier for the single active constraint from (30).

Step 7 Derive $\boldsymbol{\lambda}^e$ from equations

$$\text{if } u \text{ is active:} \qquad \mathbf{K}(n)\cdot\boldsymbol{\lambda}^e - \boldsymbol{\lambda}^d = \mathbf{0},$$
$$\text{if } \sigma \text{ is active:} \qquad \mathbf{K}(n)\;\boldsymbol{\lambda}^e - \boldsymbol{\lambda}^s\mathbf{EB} = \mathbf{0}.$$

Step 8 Input obtained design variables values to $\partial L/\partial A$ equation and find residuals $D_j(n)$. If residual is small enough end computations, else proceed further.

Step 9 $n := n+1$.

Step 10 Find new design variables

$$A_j(n+1) := A_j(n)\left[1 + \beta\frac{D_j(n)j_0}{\left\{\sum_{j=1}^{j_0}\left[D_j(n)\,A_j(n)\right]^2\right\}^{0.5}}\right]$$

Step 11 If $A(n+1) < A^{\min}$ then $A(n+1) := A^{\min}$.

Step 12 Go to 3.

The presented above method and the appropriate algorithm have some disadvantages. The main one is that the tolerances are considered only during determining scaling factor r – see step 4. They are not included in other iteration steps i.e. 3, 7, although some variables, for instance matrix $\mathbf{K}$, are functions of truss dimensions (A_j, l_j). This approximation could be done because at all iteration steps the error of $\mathbf{K}$ estimation caused by the lengths' tolerance remains constant (members' lengths are not changing at successive approaches) and moreover, the scaling factor is very close to 1 (especially at the end of calculation process).

The mentioned estimation error is partially compensated by the consideration of products of absolute values of members' tolerances and appropriate sensitivities i.e. the worst possible case is taken into account – see (15), (16), (29) etc.

6. Numerical example

As a test example, a typical 10-bar bench-mark structure was considered [4, 5, 6, 7]. The system is subjected to the displacement constraint $u^0 = 2.0$ in (0.0508 m) on all nodal displacements and the stress constraint with the maximum allowable tension $\sigma^0 = 2.5\text{e}+04$ lbsi (172.37 MPa) imposed on all structural members. The minimum size $A = 0.1$ in^2 (0.000064 m^2) was assumed. Because each cross-section is considered to be a single separate design variable, therefore, we come to the problem with 10 design variables, 8 degrees of freedom and 28 constraints (8 displacements, 10 stresses and 10 minimum cross-sections). External load of $P = 1.0\text{e}+05$ lb (444.819 kN) value is applied in nodes no. 2 and 4. Modulus of elasticity $E = 10^7$ lb/in^2 (68947.2 MPa), material density $\rho = 0.1$ lb/in^3 (27.68 kg/m^3), members' length $l = 360$ in (9.14 m). Structure's dimensions, elements' numeration, material properties and coordinate systems are put in Fig. 1.

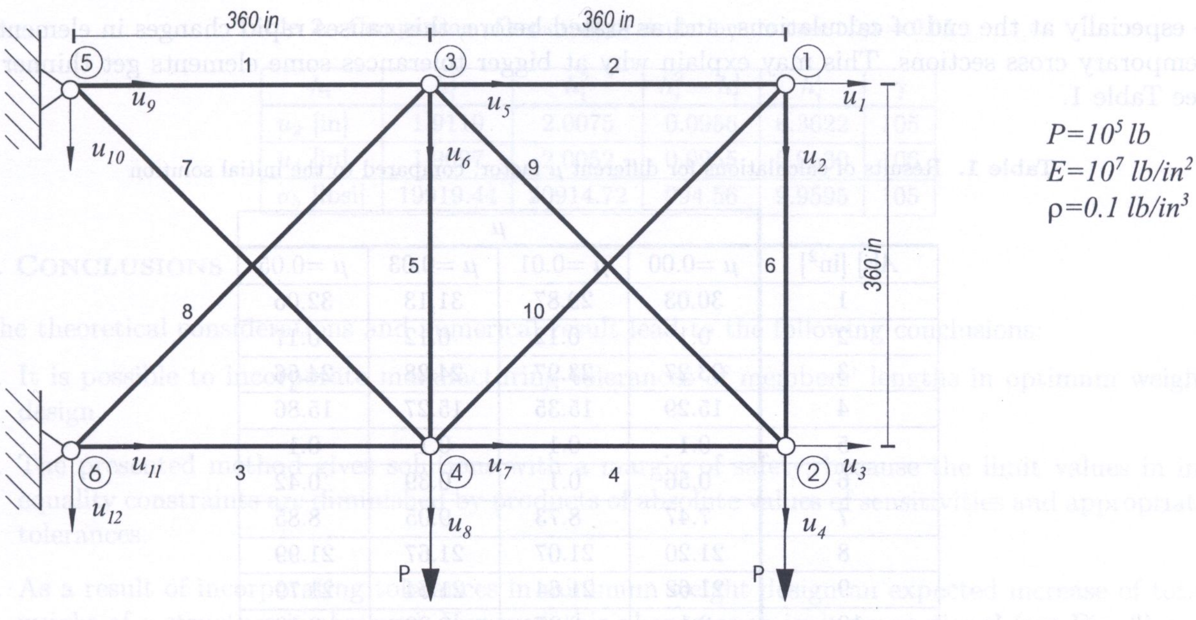

Fig. 1. The ten-bar bench-mark problem for the test analysis

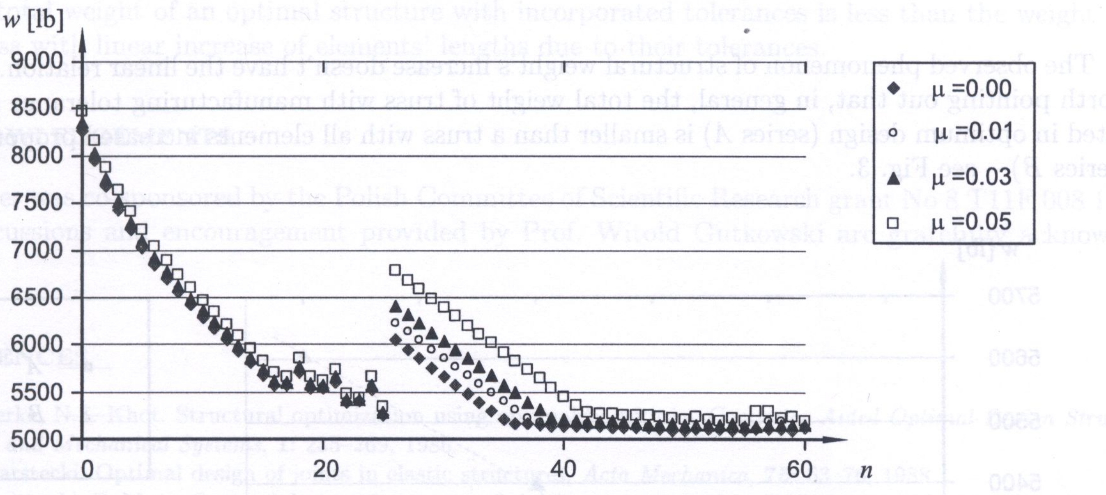

Fig. 2. Course of weight changes during successive iterations

The presented algorithm gives an optimal solution after about 25 to 60 iterations, depending mainly on the initial solution, tolerance μ, scaling factor β etc. At the beginning the structure weight goes down rapidly, but after 20–30 iterations the weight is rather oscillating then significantly decreasing. The successive solutions are there very "unstable" (Fig. 2) and some design variables may change rapidly. This could be related to the huge cross-section distribution (the ratio of the biggest to the smallest exceeds just 300) and the fact that at the end of iterations two or even three constraints come close to their limit values. In such situation it's necessary to change the scaling factor β to make the next approximations of A "finer". The observed little increase at certain iterations (i.e. 18, 24) is related to the change of a temporary active constraint.

The course of calculations is presented in the Fig. 2.

The table below presents the results of optimization problems obtained for three different tolerances $\mu = 0.01$, $\mu = 0.03$ and $\mu = 0.05$. The obtained solutions were compared to the well-known solution of $\mu = 0.00$ [8].

An expected growth of total structural weight can be notified while increasing tolerance factor μ. But incorporating manufacturing tolerances changes active constraints at certain iterations

– especially at the end of calculations, and as stated before, this causes rapid changes in elements' temporary cross sections. This may explain why at bigger tolerances some elements get thinner – see Table 1.

Table 1. Results of calculations for different μ factor, compared to the initial solution

	μ			
$A[i]$ [in^2]	μ =0.00	μ =0.01	μ =0.03	μ =0.05
1	30.03	29.87	31.13	32.05
2	0.1	0.12	0.12	0.17
3	23.27	23.97	24.28	24.66
4	15.29	15.35	15.27	15.86
5	0.1	0.1	0.1	0.1
6	0.56	0.1	0.39	0.42
7	7.47	8.73	9.05	8.85
8	21.20	21.07	21.67	21.99
9	21.62	21.64	21.64	21.79
10	0.1	0.27	0.20	0.18
Weight [lb]	**5061.6**	**5136.15**	**5243.04**	**5326.02**

The observed phenomenon of structural weight's increase doesn't have the linear relation. But it's worth pointing out that, in general, the total weight of truss with manufacturing tolerance incorporated in optimum design (series A) is smaller than a truss with all elements increased proportionally (series B) – see Fig. 3.

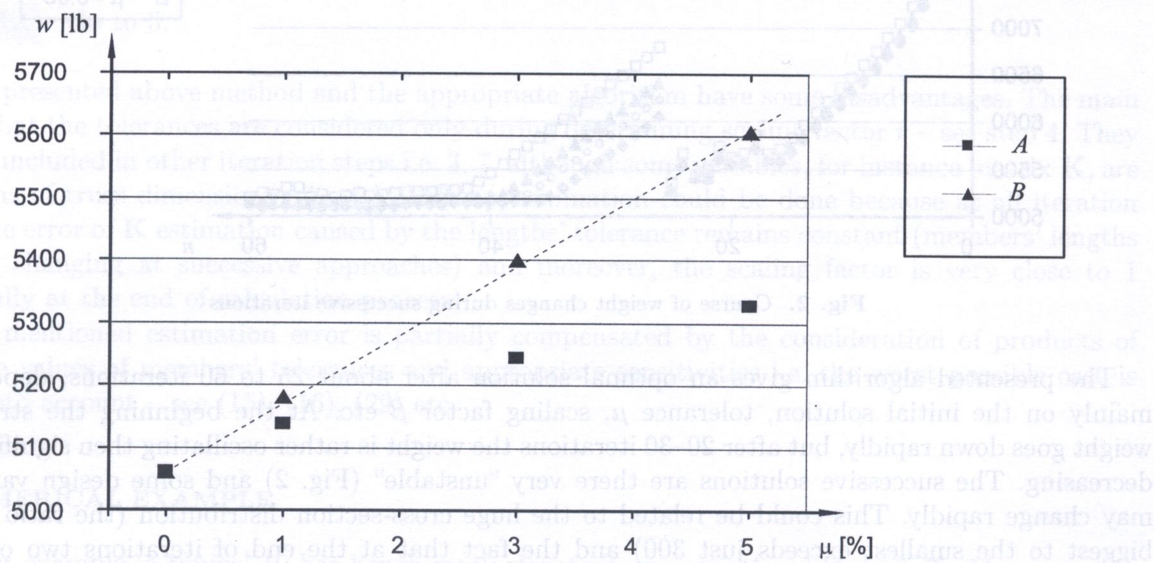

Fig. 3. Obtained solutions (series A) in comparison to the linear design variables change (series B)

Finally, the presented solutions were compared to the results obtained by searching for optimum design in initial and in the modified problem – i.e. all dimensions were changed proportionally $l := l + \Delta l$. Let h_i^1 denote the initial value of a h_i constraint, h_i^2 denote the same constraint value for modified design, and h_i' denote the change of constraint predicted by the sensitivity formula i.e. $h_i' = \sum_{j=1}^{j_0} |s_{ij}| \mu l_j$. The measure of the derivative accuracy is a ratio γ equal $(h_i^2 - h_i^1)/h_i' \cdot 100\%$. All results are gathered in Table 2.

Table 2. Comparison of sensitivity calculations in case of $\mu = 0.05$

h_i	h_i^1	h_i^2	$h_i^2 - h_i^1$	h_i'	γ
u_2 [in]	1.9119	2.0075	0.0956	0.3622	105
u_4 [in]	1.9097	2.0052	0.0955	0.0530	106
σ_5 [lbsi]	19919.44	20914.72	994.56	9.9595	105

7. Conclusions

The theoretical considerations and numerical result lead to the following conclusions:

1. It is possible to incorporate manufacturing tolerances of members' lengths in optimum weight design.
2. The presented method gives solutions with a margin of safety, because the limit values in inequality constraints are diminished by products of absolute values of sensitivities and appropriate tolerances.
3. As a result of incorporating tolerances in minimum weight design an expected increase of total weight of a structure is observed. However, this phenomenon isn't proportional (see Fig. 3) and moreover, some cross sections may decrease.
4. The total weight of an optimal structure with incorporated tolerances is less than the weight of a truss with linear increase of elements' lengths due to their tolerances.

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