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On the stability of Jeffcott rotor in fluid-film bearings[1]

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For some combinations of rotor speed and radial load, the pressure field of bearing fluid can perturb the pure rotational motion and disturb the normal operation of a rotating machine. Classical approach to the stability analysis of Jeffcott rotor in fluid-film bearings is modelling bearings as spring-damper elements and disregarding the external rotor damping [1, 5]. Nonlinear models are used to verify results obtained from a linearized model.

This paper deals with the influence of external rotor damping on the size of stability regions. Stability analysis of the Jeffcott rotor in fluid-film bearings is performed by using both the linear model based on the linearization of bearing force around the static equilibrium position and the nonlinear model of the velocity linearization [2, 3].

1. Introduction

It is a well-known fact that, for some combinations of rotor speed and radial load, rotors horizontally supported by fluid-film bearings can develop an unstable behavior. This phenomenon is intrinsically linked with the fluid-film action within the bearings. The internal feedback mechanism transfers the part of rotational energy of the bearing fluid into self-excited vibrations called whirling. The whirling is the precessional motion of the rotor at its natural frequency.

The behavior of a rotating machine is stable if its shaft performs purely rotational motion around an eccentric axis within the bearing (static equilibrium position) at a required rotational speed, and no random perturbation can drastically change its behavior [4, 9]. The stability analysis of a rotor in fluid-film bearings is inherently a nonlinear problem because the hydrodynamic bearing forces are strongly nonlinear function of the relative journal displacement and velocity. A linear approach to the stability analysis concerns linearization of the bearing force in the vicinity of the equilibrium position. The nonlinear stability analysis is based on different models of solid/fluid interaction phenomena and requires long numerical computations.

The objective of this paper is to investigate the influence of external rotor damping on the stability of Jeffcott rotor in fluid-film bearings, considering both the linear and nonlinear approach. A nonlinear approach is based on the Crandall model of the velocity linearization [2, 3] that allows a relatively simple numerical stability analysis without very tiring and time consuming integration used in more sophisticated bearing models. The results show that the Crandall procedure and the linearization of bearing force are in a very good agreement.

2. Jeffcott rotor on fluid-film bearings

The model of Jeffcott rotor in fluid-film bearings is used to study the instability fields of rotors. The model consists of a rigid disc of mass m attached to a massless flexible shaft. The only force

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acting on the disc is that due to the stiffness k of the shaft. The shaft is horizontally situated and supported in a pair of identical fluid-film bearings whose clearance is c as shown in Fig. 1. In the study of the flexural behavior, the shaft rotates with constant angular velocity Ω and undergoes transverse motion only.

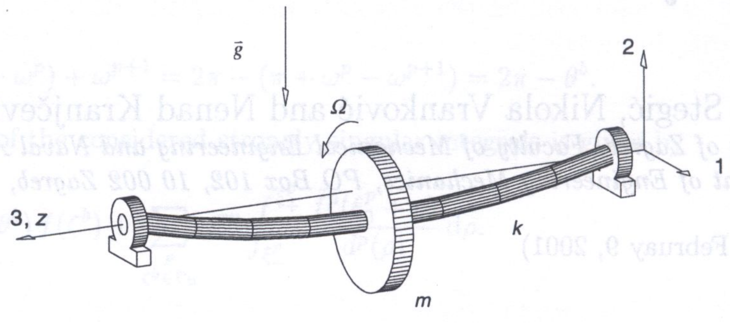

Fig. 1. Model of Jeffcott rotor in fluid-film bearings

With $\mathbf{x} = \begin{bmatrix} x_1/c \\ x_2/c \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1/c \\ y_2/c \end{bmatrix}$ being the nondimensional displacements of rotor and journal, respectively, the equation of motion for rotor can be expressed in nondimensional form

$$\eta^2 \mathbf{x}'' + \eta\zeta_r \mathbf{x}' + (\mathbf{x} - \mathbf{y}) = \mathbf{f}_\Gamma \,, \tag{1}$$

where prime denotes the differentiation with respect to $\tau = \Omega t$, $\eta = \Omega\sqrt{m/k}$ is the nondimensional rotor speed, $\zeta_r = c_r/\sqrt{mk}$ denotes the nondimensional external rotor damping factor and $\mathbf{f}_\Gamma = \begin{bmatrix} 0 \\ 2\Gamma \end{bmatrix}$ is the vector of the nondimensional radial load. If the radial load is simply the weight of rotor, then $\Gamma = \frac{mg}{2kc}$.

The equilibrium requirement at each bearing yields

$$-(\mathbf{x} - \mathbf{y})/2 + \mathbf{f}_H = \mathbf{0}, \tag{2}$$

where $\mathbf{f}_H$ is the vector of hydrodynamic journal force in nondimensional form.

3. Journal forces exerted by the fluid-film

The fluid-film bearing is sketched in Fig. 2. The cylindrical journal of radius R turns in the fluid-film bore of radius $R+c$ and length L. The journal is statically or dynamically loaded in the radial direction, and its position with respect to the center of bearing is defined by the eccentricity e and the attitude angle γ.

The transverse motion of the journal disturbs the fluid flow in the clearance gap by creating large local pressure changes within the bearing fluid. Because the clearance ratio c/R is generally of the order of 10^{-3}, the pressure p is linked to the thickness h of the fluid-film as per Reynolds equation. The Reynolds equation (in terms of cylindrical coordinates), applied on isoviscous and incompressible Newtonian fluid operating in the laminar regime, has the form

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial\varphi}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6\Omega\frac{\partial h}{\partial\varphi} + 12\frac{\partial h}{\partial t}, \tag{3}$$

where μ is the absolute viscosity of the bearing fluid. The film thickness h is easily expressed as a function of the journal position,

$$h = c - e\cos\varphi. \tag{4}$$

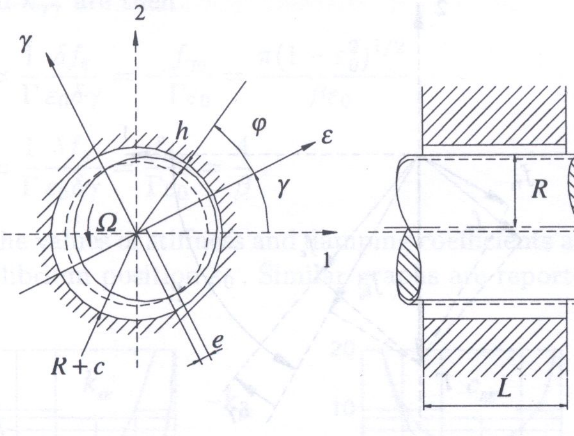

Fig. 2. Clearance geometry of fluid-film cylindrical bearing

The Reynolds equation (3) can be used to obtain the pressure distribution in the fluid-film. If the bearing is relatively short, the flow in the circumferential direction may be neglected and the short bearing solution obtained [10]. By neglecting the term linked with the circumferential pressure change of the Reynolds equation (3), the pressure distribution in the fluid-film can be obtained from

$$p(\varphi, z) = \frac{6\mu}{h^3}\left[\dot{e}\cos\varphi + e\left(\dot{\gamma} - \frac{\Omega}{2}\right)\sin\varphi\right]\left(\frac{L^2}{4} - z^2\right) \tag{5}$$

with the boundary condition $z = \pm\frac{L}{2}$, $p = 0$.

The pressure is positive between angles φ_1 and φ_2 defined by $\varphi_1 = \frac{\pi}{2} + \alpha$ and $\varphi_2 = \frac{3\pi}{2} + \alpha$, respectively, where $\alpha = \operatorname{atan}\frac{e\Omega/2 - \dot{\gamma}}{\dot{e}} \geq \frac{\pi}{2}$.

The bearing force components acting on the journal, parallel and normal to the eccentricity vector, can be obtained by integrating the pressure distribution (5) on the positive portion of the fluid-film only i.e., $[\varphi_1\varphi_2]$,

$$F_\varepsilon = \int_{\varphi_1}^{\varphi_2}\int_{-\frac{L}{2}}^{\frac{L}{2}} p(\varphi, z)\cos\varphi\, R\,\mathrm{d}\varphi\,\mathrm{d}z = \frac{\pi\mu R L^3}{c^3}\left[\frac{1+2\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}\dot{e} - \frac{2\varepsilon}{\pi(1-\varepsilon^2)^2}e\left(\dot{\gamma} - \frac{\Omega}{2}\right)\right], \tag{6}$$

$$F_\gamma = \int_{\varphi_1}^{\varphi_2}\int_{-\frac{L}{2}}^{\frac{L}{2}} p(\varphi, z)\sin\varphi\, R\,\mathrm{d}\varphi\,\mathrm{d}z = \frac{\pi\mu R L^3}{c^3}\left[-\frac{2\varepsilon}{\pi(1-\varepsilon^2)^2}\dot{e} + \frac{1}{2(1-\varepsilon^2)^{3/2}}e\left(\dot{\gamma} - \frac{\Omega}{2}\right)\right], \tag{7}$$

where $\varepsilon = e/c$ denotes the nondimensional eccentricity.

When using the nondimensional time differentiation, the journal force components take the form

$$f_\varepsilon = \zeta_B\eta\left[\frac{\varepsilon^2}{\pi(1-\varepsilon^2)^2} + \frac{1+2\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}\varepsilon' - \frac{2\varepsilon^2}{\pi(1-\varepsilon^2)^2}\gamma'\right], \tag{8}$$

$$f_\gamma = \zeta_B\eta\left[-\frac{\varepsilon}{4(1-\varepsilon^2)^{3/2}} - \frac{2\varepsilon}{\pi(1-\varepsilon^2)^2}\varepsilon' + \frac{\varepsilon}{2(1-\varepsilon^2)^{3/2}}\gamma'\right], \tag{9}$$

where $\zeta_B = \pi\mu R L^3/(c^3\sqrt{mk})$ is the bearing factor in nondimensional form.

4. Linearization of the journal forces

Determination of the journal static displacement is essential for the linearization procedure. The kinematic requirement for equilibrium is that the journal velocity relative to the bearing, vanishes; i.e., $\varepsilon' = \gamma' = 0$. To define the static journal displacement, the applied static load Γ (without loss

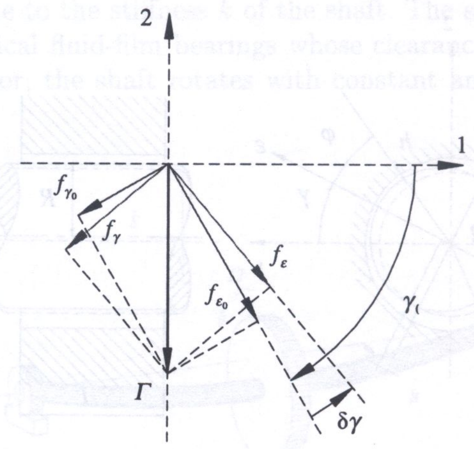

Fig. 3. Equilibrium condition for a fluid-film bearing

of generality) is directed along the vertical axis as shown in Fig. 3. The eccentricity magnitude ε_0 and the attitude angle γ_0 at the equilibrium position are obtained as follows,

$$\Gamma \quad = \zeta_B \eta \frac{\varepsilon_0 \beta}{4\pi(1-\varepsilon_0^2)^2}\,, \tag{10}$$

$$\tan\gamma_0 = -\frac{4\varepsilon_0}{\pi(1-\varepsilon_0^2)^{1/2}}\,, \tag{11}$$

where $\beta = [16\varepsilon_0^2 + \pi^2(1-\varepsilon_0^2)]^{1/2}$.

Assuming small displacement of the journal center from the equilibrium position, $\delta\varepsilon$ and $\delta\gamma$, the journal force components in the polar coordinates (8) and (9) may be linearized around the static equilibrium position to obtain

$$f_\varepsilon = f_{\varepsilon 0} + \Gamma k_{\varepsilon\varepsilon}\delta\varepsilon + \Gamma k_{\varepsilon\gamma}\varepsilon_0\delta\gamma + \Gamma c_{\varepsilon\varepsilon}\delta\varepsilon' + \Gamma c_{\varepsilon\gamma}\varepsilon_0\delta\gamma'\,, \tag{12}$$

$$f_\gamma = f_{\gamma 0} + \Gamma k_{\gamma\varepsilon}\delta\varepsilon + \Gamma k_{\gamma\gamma}\varepsilon_0\delta\gamma + \Gamma c_{\gamma\varepsilon}\delta\varepsilon' + \Gamma c_{\gamma\gamma}\varepsilon_0\delta\gamma'\,, \tag{13}$$

where $f_{\varepsilon 0}$ and $f_{\gamma 0}$ are the journal force components under the static conditions while ($k_{\varepsilon\varepsilon}$, $k_{\varepsilon\gamma}$, $k_{\gamma\varepsilon}$, $k_{\gamma\gamma}$) and ($c_{\varepsilon\varepsilon}$, $c_{\varepsilon\gamma}$, $c_{\gamma\varepsilon}$, $c_{\gamma\gamma}$) are the stiffness and damping coefficients. The coefficients $k_{\varepsilon\varepsilon}$, $k_{\gamma\varepsilon}$, $c_{\varepsilon\varepsilon}$, $c_{\varepsilon\gamma}$, $c_{\gamma\varepsilon}$ and $c_{\gamma\gamma}$ can be derived directly from Eqs. (8) and (9) as follows,

$$k_{\varepsilon\varepsilon} = \frac{1}{\Gamma}\frac{\partial f_\varepsilon}{\partial\varepsilon}\bigg|_{\varepsilon_0,\gamma_0} = \frac{8(1+\varepsilon_0^2)}{\beta(1-\varepsilon_0^2)}\,, \qquad k_{\gamma\varepsilon} = \frac{1}{\Gamma}\frac{\partial f_\gamma}{\partial\varepsilon}\bigg|_{\varepsilon_0,\gamma_0} = -\frac{\pi(1+2\varepsilon_0^2)}{\beta\varepsilon_0(1-\varepsilon_0^2)^{1/2}}\,, \tag{14}$$

$$c_{\varepsilon\varepsilon} = \frac{1}{\Gamma}\frac{\partial f_\varepsilon}{\partial\varepsilon'}\bigg|_{\varepsilon_0,\gamma_0} = \frac{2\pi(1+2\varepsilon_0^2)}{\beta\varepsilon_0(1-\varepsilon_0^2)^{1/2}}\,, \qquad c_{\varepsilon\gamma} = \frac{1}{\Gamma}\frac{\partial f_\varepsilon}{\varepsilon\partial\gamma'}\bigg|_{\varepsilon_0,\gamma_0} = -\frac{8}{\beta}\,, \tag{15}$$

$$c_{\gamma\varepsilon} = \frac{1}{\Gamma}\frac{\partial f_\gamma}{\partial\varepsilon'}\bigg|_{\varepsilon_0,\gamma_0} = -\frac{8}{\beta}\,, \qquad c_{\gamma\gamma} = \frac{1}{\Gamma}\frac{\partial f_\gamma}{\varepsilon\partial\gamma'}\bigg|_{\varepsilon_0,\gamma_0} = \frac{2\pi(1-\varepsilon_0^2)^{1/2}}{\beta\varepsilon_0}\,. \tag{16}$$

The stiffness coefficients $k_{\varepsilon\gamma}$ and $k_{\gamma\gamma}$ may be obtained by considering the consequences of a perturbation $\delta\gamma$ of the equilibrium angle γ_0 with ε_0 and Γ maintained constant. The rotation $\delta\gamma$ gives the following restatement of equilibrium force (see Fig. 3),

$$f_\varepsilon = f_{\varepsilon 0}\cos(\delta\gamma) - f_{\gamma 0}\sin(\delta\gamma) \approx f_{\varepsilon 0} - f_{\gamma 0}\delta\gamma = f_{\varepsilon 0} + \delta f_\varepsilon\,, \tag{17}$$

$$f_\gamma = f_{\varepsilon 0}\sin(\delta\gamma) + f_{\gamma 0}\cos(\delta\gamma) \approx f_{\gamma 0} + f_{\varepsilon 0}\delta\gamma = f_{\gamma 0} + \delta f_\gamma\,. \tag{18}$$

The coefficients $k_{\varepsilon\gamma}$ and $k_{\gamma\gamma}$ are then

$$k_{\varepsilon\gamma} = \frac{1}{\Gamma}\frac{\partial f_\varepsilon}{\varepsilon\partial\gamma}\bigg|_{\varepsilon_0,\gamma_0} \approx \frac{1}{\Gamma}\frac{\delta f_\varepsilon}{\varepsilon_0\delta\gamma} = -\frac{f_{\gamma 0}}{\Gamma\varepsilon_0} = \frac{\pi(1-\varepsilon_0^2)^{1/2}}{\beta\varepsilon_0}, \tag{19}$$

$$k_{\gamma\gamma} = \frac{1}{\Gamma}\frac{\partial f_\gamma}{\varepsilon\partial\gamma}\bigg|_{\varepsilon_0,\gamma_0} \approx \frac{1}{\Gamma}\frac{\delta f_\gamma}{\varepsilon_0\delta\gamma} = \frac{f_{\varepsilon 0}}{\Gamma\varepsilon_0} = \frac{4}{\beta}. \tag{20}$$

Figure 4 illustrates the values of stiffness and damping coefficients as a function of nondimensional eccentricity at the equilibrium position ε_0. Similar graphs are reported in [7, 8, 12]

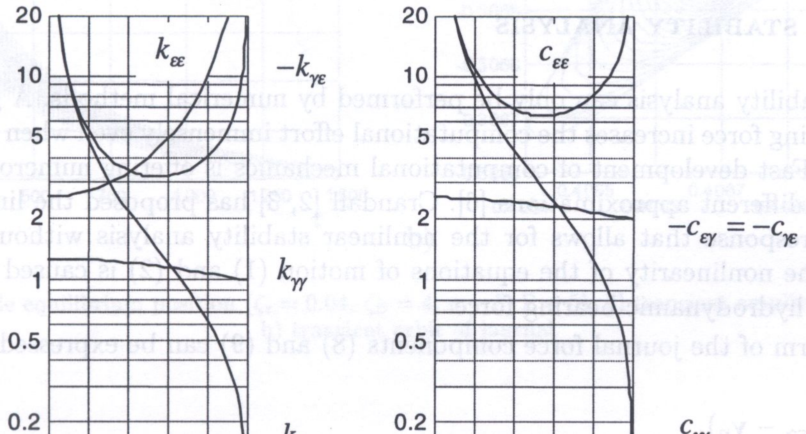

Fig. 4. Stiffness and damping coefficients in the polar coordinates as a function of ε_0

5. Linear stability analysis

The linearized journal force (12) and (13) can be written in the matrix form as

$$\mathbf{f}_{\varepsilon\gamma} = \mathbf{f}_{\varepsilon_0\gamma_0} + \Gamma\mathbf{K}_{\varepsilon\gamma}\,\delta\mathbf{u} + \Gamma\mathbf{C}_{\varepsilon\gamma}\,\delta\mathbf{u}', \tag{21}$$

where $\mathbf{f}_{\varepsilon\gamma} = \begin{bmatrix} f_\varepsilon \\ f_\gamma \end{bmatrix}$ is the vector of journal force in the polar coordinates, $\mathbf{K}_{\varepsilon\gamma} = \begin{bmatrix} k_{\varepsilon\varepsilon} & k_{\varepsilon\gamma} \\ k_{\gamma\varepsilon} & k_{\gamma\gamma} \end{bmatrix}$ and $\mathbf{C}_{\varepsilon\gamma} = \begin{bmatrix} c_{\varepsilon\varepsilon} & c_{\varepsilon\gamma} \\ c_{\gamma\varepsilon} & c_{\gamma\gamma} \end{bmatrix}$ are the matrices of stiffness and damping coefficients, respectively and $\delta\mathbf{u} = \begin{bmatrix} \delta\varepsilon \\ \varepsilon_0\,\delta\gamma \end{bmatrix}$ denotes the vector of small displacement from the equilibrium position.

If $\delta\mathbf{x}$ and $\delta\mathbf{y}$ are small displacements in horizontal and vertical directions from the rotor and journal equilibrium position, the governing equations of motion (1) and (2) take the form suitable for the linear stability analysis,

$$\eta^2\delta\mathbf{x}'' + \eta\zeta_r\delta\mathbf{x}' + (\delta\mathbf{x} - \delta\mathbf{y}) = \mathbf{0}, \tag{22}$$

$$-\frac{1}{2}(\delta\mathbf{x} - \delta\mathbf{y}) + \Gamma\,\mathbf{T}^{-1}\mathbf{K}_{\varepsilon\gamma}\mathbf{T}\,\delta\mathbf{y} + \Gamma\,\mathbf{T}^{-1}\mathbf{C}_{\varepsilon\gamma}\mathbf{T}\delta\mathbf{y}' = \mathbf{0}, \tag{23}$$

where $\mathbf{T} = \begin{bmatrix} \cos\gamma_0 & \sin\gamma_0 \\ -\sin\gamma_0 & \cos\gamma_0 \end{bmatrix}$ is the matrix of transformation.

Equations (22) and (23) have the following characteristic sixth-order equation,

$$b_0 + b_1\lambda + b_2\lambda^2 + b_3\lambda^3 + b_4\lambda^4 + b_5\lambda^5 + b_6\lambda^6 = 0, \tag{24}$$

in which λ is the eigenvalue and $b_0 \ldots b_6$ are the coefficients.

The behavior of Jeffcott rotor in fluid-film bearings becomes stable when all determinants of the Hurwitz matrix applied to the characteristic equation (24) are positive.

6. Nonlinear stability analysis

The nonlinear stability analysis can only be performed by numerical methods. A general nonlinear model of the bearing force increases the computational effort immensely even when the short bearing solution is used. Fast development of computational mechanics is offering numerous new nonlinear models based on different approximations [6]. Crandall [2, 3] has proposed the linearization of the journal velocity response that allows for the nonlinear stability analysis without long numerical computations. The nonlinearity of the equations of motion (1) and (2) is caused by the nonlinear properties of the hydrodynamic bearing force.

The matrix form of the journal force components (8) and (9) can be expressed as follows,

$$\mathbf{f}_{\varepsilon\gamma} = \zeta_B\, \mathbf{Z}(\eta \mathbf{v}_{\varepsilon\gamma} - \mathbf{v}_\eta), \tag{25}$$

where $\mathbf{v}_{\varepsilon\gamma} = \begin{bmatrix} \varepsilon' \\ \varepsilon\gamma' \end{bmatrix}$ and $\mathbf{v}_\eta = \begin{bmatrix} 0 \\ \varepsilon\eta/2 \end{bmatrix}$ are the vectors of bearing velocities and $\mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ is the matrix of journal displacement.

Elements of the matrix $\mathbf{Z}$ are considered from Eqs. (8) and (9) as

$$z_{11} = \frac{1+2\varepsilon^2}{2(1-\varepsilon^2)^{5/2}}, \qquad z_{22} = \frac{1}{2(1-\varepsilon^2)^{3/2}}, \qquad z_{12} = z_{21} = -\frac{2\varepsilon}{\pi(1-\varepsilon^2)^2}.$$

By introducing the transformation matrix $\mathbf{T}$ into Eq. (25), a nonlinear set of equations of motion (1) and (2) is obtained as follows,

$$\eta^2\mathbf{x}'' + \eta\zeta_r\,\mathbf{x}' + (\mathbf{x} - \mathbf{y}) = \mathbf{f}_\Gamma, \tag{26}$$

$$-\frac{1}{2}(\mathbf{x} - \mathbf{y}) + \zeta_B\eta\,\mathbf{T}^{-1}\mathbf{Z}\mathbf{T}\mathbf{y}' - \zeta_B\,\mathbf{T}^{-1}\mathbf{Z}\mathbf{v}_\eta = \mathbf{0}. \tag{27}$$

Equations (26) and (27) can only be solved numerically. By applying the classical perturbation method, the stability of Jeffcott rotor in fluid-film bearings can be analyzed for a wide range of nondimensional speeds η and nondimensional loads Γ. The stability of Eqs. (26) and (27) is examined by studying the motion in immediate neighborhood of equilibrium position by superimposing a small disturbance on the rotor velocity η. For stable combinations of nondimensional speed and load, the amplitude of the journal displacement from equilibrium position

$$\hat{\alpha} = \sqrt{(y_1 - y_{10})^2 + (y_2 - y_{20})^2}, \tag{28}$$

converges towards zero (Fig. 5a) and the journal motion settles down to a decreasing spiral centered on the equilibrium position (Fig. 5b).

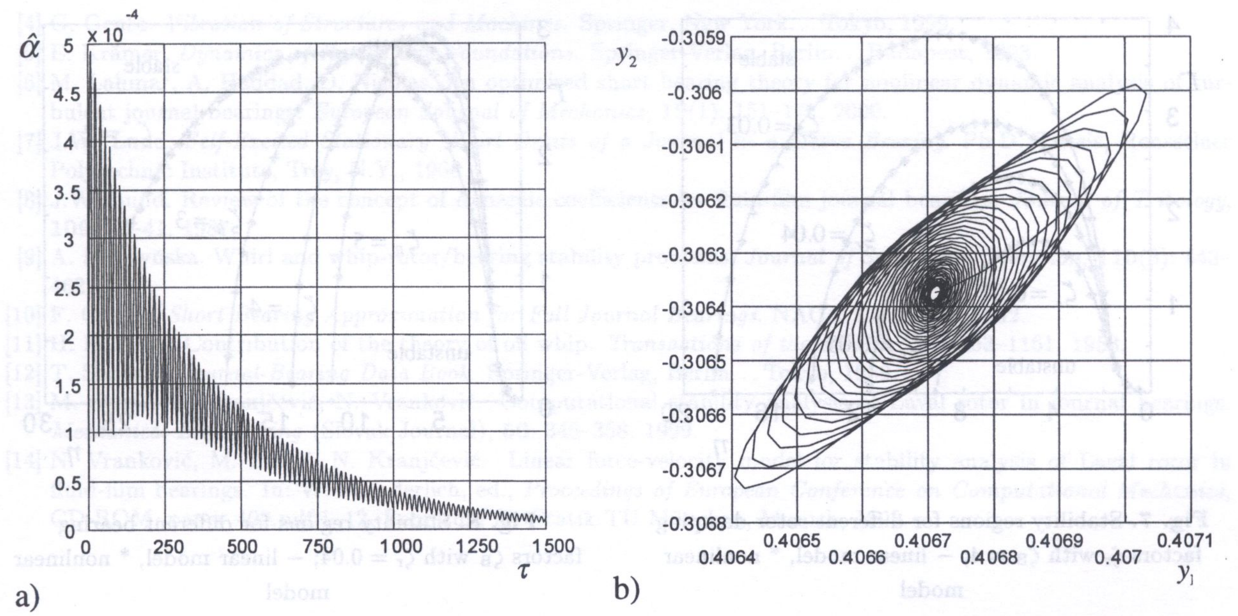

Fig. 5. Stable equilibrium position ($\zeta_r = 0.04$, $\zeta_B = 4$, $\eta = 5$, $\Gamma = 5$); a) transient amplitude of journal, b) transient orbit of journal

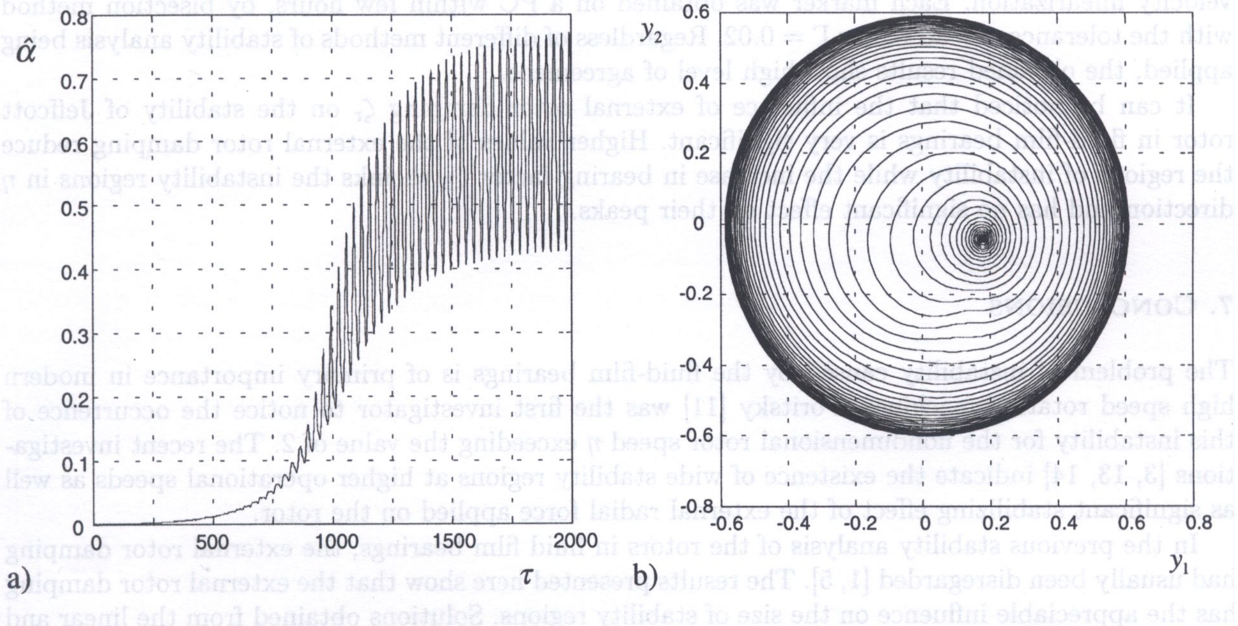

Fig. 6. Unstable equilibrium position ($\zeta_r = 0.04$, $\zeta_B = 4$, $\eta = 5$, $\Gamma = 1$); a) transient amplitude of journal, b) transient orbit of journal

For unstable combinations of η and Γ, the amplitude $\hat{\alpha}$ diverges (Fig. 6a) and the journal motion settles into an increasing spiral which converges to a stable limit cycle (Fig. 6b) or diverges as shown in [3]. In Eq. (28), (y_{10}, y_{20}) denotes the journal static displacement in a nondimensional form.

During the computations, Eqs. (26) and (27) have been integrated using the numerical routine Runge–Kutta 5. To ensure the linearization of the velocity dependence, the velocities ε' and $\varepsilon(\gamma' - \eta/2)$ were monitored throughout the computations. The value of $|\varepsilon'|$ is of an order of magnitude smaller than the value of $|\varepsilon(\gamma' - \eta/2)|$, which makes this linearization allowable. The fraction of the period over which magnitude of $|\varepsilon(\gamma' - \eta/2)|$ is comparable with magnitude of $|\varepsilon'|$ is negligible.

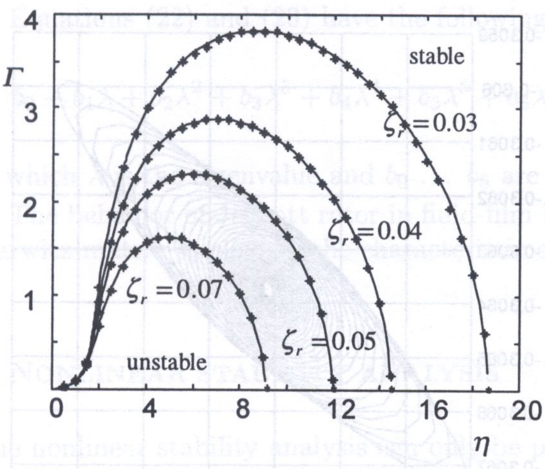

Fig. 7. Stability regions for different rotor damping factors ζ_r with $\zeta_B = 4$; – linear model, * nonlinear model

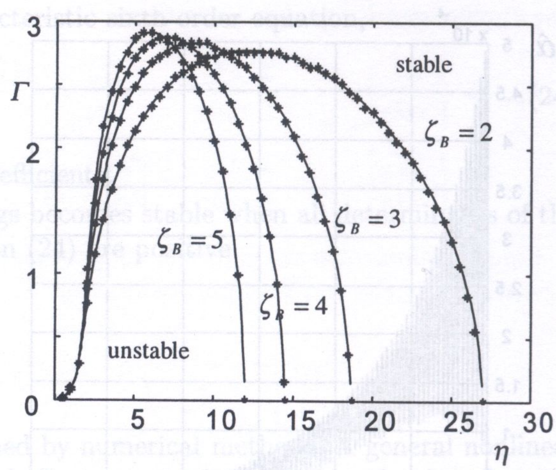

Fig. 8. Stability regions for different bearing factors ζ_B with $\zeta_r = 0.04$; – linear model, * nonlinear model

As the result of stability analyses, the regions of stable dynamic behavior of rotor (in terms of rotor speed and radial load) for different values of external rotor damping factor and bearing factor are presented in Figs. 7 and 8. Solid lines represent borderlines obtained from the linear stability analysis, while markers denote the instability thresholds obtained by the Crandall procedure of velocity linearization. Each marker was obtained on a PC within few hours, by bisection method with the tolerance of load factor $\Gamma = 0.02$. Regardless of different methods of stability analysis being applied, the obtained results show high level of agreement.

It can be noticed that the influence of external rotor damping ζ_r on the stability of Jeffcott rotor in fluid-film bearings is very significant. Higher values of the external rotor damping reduce the regions of instability while the increase in bearing factor ζ_B shrinks the instability regions in η direction and has no significant effect on their peaks.

7. Conclusions

The problem of instability caused by the fluid-film bearings is of primary importance in modern high speed rotating machines. Poritsky [11] was the first investigator to notice the occurrence of this instability for the nondimensional rotor speed η exceeding the value of 2. The recent investigations [3, 13, 14] indicate the existence of wide stability regions at higher operational speeds as well as significant stabilizing effect of the external radial force applied on the rotor.

In the previous stability analysis of the rotors in fluid film bearings, the external rotor damping had usually been disregarded [1, 5]. The results presented here show that the external rotor damping has the appreciable influence on the size of stability regions. Solutions obtained from the linear and nonlinear models are consistent. The linearized model is suitable for the stability analysis over a wide range of the external rotor damping parameters. Numerical calculations based on the nonlinear model verify the validity of results from the linearized model.

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