Computer Assisted Mechanics and Engineering Sciences, 10: 81–91, 2003.

Localization of plastic deformations as a result of wave interaction

Adam Glema and Tomasz Łodygowski
Institute of Structural Engineering, Poznań University of Technology, Piotrowo 5, 60-965 Poznań, Poland

Piotr Perzyna
Institute of Fundamental Technological Research, Polish Academy of Sciences, Świętokrzyska 21, 00-049 Warsaw, Poland

(Received January 26, 2001)

The main objective of the paper is the investigation of the interaction and reflection of elastic-viscoplastic waves which can lead to localization phenomena in solids. The rate type constitutive structure for an elastic-viscoplastic material with thermomechanical coupling is used. An adiabatic inelastic flow process is considered. Discussion of some features of rate dependent plastic medium is presented. This medium has dissipative and dispersive properties. In the evolution problem considered in such dissipative and dispersive medium the stress and deformation due to wave reflections and interactions are not uniformly distributed, and this kind of heterogeneity can lead to strain localization in the absence of geometrical or material imperfections.

Numerical examples are presented for a 2D specimens subjected to tension, with the controlled displacements imposed at one side with different velocities. The initial-boundary conditions which are considered reflect the asymmetric (single side) tension of the specimen with the opposite side fixed, which leads to non-symmetric deformation. The influence of the constitutive parameter (relaxation time of mechanical perturbances) is also studied in the examples. The attention is focused on the investigation of the interactions and reflections of waves and on the location of localization of plastic deformations.

1. Introduction

The description of the structural damage process bases on dynamic experiments with controlled load amplitudes and time duration. Failure of solids is as a highly rate, temperature and history dependent, nonlinear process. The process of deformation is formulated by the set of evolution equations for velocity, mass, Kirchhoff stresses, the microdamage initiation and growth, and temperature within the theory of viscoplasticity. The finite element method is applied for solution of the problem. It is known and discussed in many papers that, classical rate independent plastic strain formulation with negative stress–strain constitutive relation (softening) leads to ill-posed problems and in consequence to not unique results in numerical applications (for example see Łodygowski and Perzyna [11]). From theoretical point of view we observe the change of the type of partial differential equations (elliptic into hyperbolic in statics and conversely in dynamics) and in consequence from numerical point of view we observe parasitic mesh sensitivity (e.g. Łodygowski [9]). The regularization is the way to avoid these phenomena. The rate dependent plasticity and dynamic formulation are used in the presentation to describe the localization phenomenon including the influence of temperature and microdamage effects. In the evolution problem (dynamic formulation) introducing the Perzyna's type viscoplasticity, naturally regularizes initial boundary value problems. Then, the governing equations do not change the type during the strain localization damage up to failure and the process can be studied as a well posed problem. As a crucial effect the solution becomes unique, the results are stable and spurious mesh dependency is not observed any more.

The regularization introduced by viscoplasticity (relaxation time plays also the role of regularization parameter) is the way to avoid ill-posedness of the formulation.

The fracture phenomenon can occur as a result of an adiabatic shear band localization attributed to a plastic instability implied by microdamage and thermal softening during dynamic plastic flow processes.

The correspondence between stationary body waves and bulk localization has long been appreciated (cf. Hadamard [7], Thomas [33], Hill [8], Truesdell [34], Mandel [13] and Rice [27]). In many recently published papers the investigation of adiabatic shear band localization phenomena has been based on an analysis of acceleration waves and has taken advantage of a notion of the instantaneous adiabatic acoustic tensor (cf. Rice [27], Ottosen and Runesson [19], Duszek-Perzyna and Perzyna [2, 3] and Perzyna [22]). Connection between stationary waves, stability and well-posedness of initial-boundary value problems has received considerable attention (cf. Simpson and Spector [28], Dowaikh and Ogden [1] and Ogden [18]). The analysis of the influence of the effect of boundaries and interfaces on shear band localization in time- and rate-independent plastic materials has been based on the investigation of stationary body, Rayleigh and Stoneley waves (cf. Needleman and Ortiz [16] and Suo, Ortiz and Needleman [31]). They defined stability, in the sense of limits to the uniqueness of solutions to quasi-static boundary value problems and addressed stability in terms of the existence of certain stationary waves.

Very recently, it has been widely recognized to consider an elastic-viscoplastic model of a material as a regularization method for solving mesh dependent strain softening problems of plasticity. In these regularized initial-boundary value problems wave propagation phenomena play a fundamental role. Since an elastic-viscoplastic model introduces dissipative as well as dispersive nature for the propagated waves hence the analysis of dispersive, dissipative waves and particularly their interactions and reflections have to be considered as the most important problem (cf. Needleman [14, 15], Prevost and Loret [26], Sluys [29], Sluys et al. [30], Nemes and Eftis [17], Łodygowski et al. [10], Łodygowski [9], Łodygowski and Perzyna [11, 12], Perzyna [23, 24], Perzyna and Duszek-Perzyna [25], Wang [35], Wang et al. [36] and Glema et al. [5, 6].

The main objective of the present paper is the investigation of the interaction and reflection of elastic-viscoplastic waves which can lead to localization of plastic deformations in solids.

2. ASSUMPTIONS

Let us introduce the rate type constitutive structure for an elastic-viscoplastic material in which the effects of thermomechanical couplings are taken into consideration.

The axioms are as follows:

1. Axiom of the existence of the free energy function in the form [4]

$$\psi = \hat{\psi}(\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}), \tag{1}$$

where $\mathbf{e}$ is the Eulerian strain tensor, $\mathbf{F}$ the deformation gradient, ϑ a temperature field and $\boldsymbol{\mu}$ denotes the internal state variable vector.

2. Axiom of objectivity (spatial covariance). The constitutive structure should be invariant with respect to any diffeomorphism $\boldsymbol{\xi} : \mathcal{S} \to \mathcal{S}$, where $\mathcal{S}$ denotes the actual (spatial) configuration of a body $\mathcal{B}$.

3. The axiom of entropy production. For any regular process ϕ_t, ϑ_t, $\boldsymbol{\mu}_t$ of a body $\mathcal{B}$ the constitutive functions are assumed to satisfy the reduced dissipation inequality

$$\frac{1}{\rho_{\mathrm{Ref}}} \boldsymbol{\tau} : \mathbf{d} - (\eta\dot{\vartheta} + \dot{\psi}) - \frac{1}{\rho\vartheta} \mathbf{q} \cdot \operatorname{grad}\vartheta \geq 0, \tag{2}$$

where ρ and ρ_{Ref} denote the mass density in the actual and reference configuration, respectively, $\boldsymbol{\tau}$ is the Kirchhoff stress tensor, $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$ the rate of total deformation, η denotes the specific (per unit mass) entropy and $\mathbf{q}$ is the heat vector field.

Let us also postulate $\boldsymbol{\mu} = \epsilon^p$, where $\epsilon^p = \int_0^t \left(\frac{2}{3}\mathbf{d}^p : \mathbf{d}^p\right)^{\frac{1}{2}} dt$ is the scalar measure of equivalent plastic deformation. It is introduced as the internal state variable to describe the dissipation effects generated by viscoplastic flow phenomena.

Let us assume the plastic potential function for a material in the form

$$f = J_2, \quad \text{where} \quad J_2 = \frac{1}{2}\tau'^{ab}\tau'^{cd} g_{ac} g_{bd}, \tag{3}$$

and $\mathbf{g}$ denotes the metric tensor in $\mathcal{S}$.

The evolution equation is postulated as

$$\mathbf{d}^p = \Lambda \mathbf{P}, \tag{4}$$

where for the elastic-viscoplastic model of a material we assume (cf. Perzyna [20, 21, 23, 24])

$$\Lambda = \frac{1}{T_m}\left\langle \Phi\left(\frac{f}{\kappa} - 1\right)\right\rangle, \tag{5}$$

where T_m denotes the relaxation time for mechanical disturbances and κ is the isotropic work-hardening parameter, Φ is the empirical overstress function and the bracket $\langle\cdot\rangle$ defines the ramp function, $\mathbf{P} = \frac{1}{2\sqrt{J_2}}\frac{\partial f}{\partial \boldsymbol{\tau}}$. Thus, we have

$$P_{ab} = \frac{1}{2\sqrt{J_2}}\tau'^{cd} g_{ca} g_{db}\,. \tag{6}$$

The isotropic hardening-softening material function κ is assumed in the form as follows

$$\kappa = \kappa_0^2 \left\{q + (1-q)\exp[-h(\vartheta)\epsilon^p]\right\}^2 (1 - b\vartheta), \tag{7}$$

where $q = \frac{\kappa_1}{\kappa_0}$, κ_0 and κ_1 denote the yield and saturation stress of the matrix material, respectively, $h = h(\vartheta)$ is the temperature dependent strain hardening function for the matrix material and b is a material coefficient. The overstress viscoplastic function Φ is postulated in the known form (cf. Perzyna [20, 21])

$$\Phi\left(\frac{f}{\kappa} - 1\right) = \left(\frac{f}{\kappa} - 1\right)^m, \quad \text{where } m = 1, 3, 5, \ldots \tag{8}$$

The axioms 1, 2, 3, and the evolution equations (4) lead to the rate equations as follows

$$\begin{aligned} \mathrm{L}_\upsilon \boldsymbol{\tau} &= \mathcal{L}^e : \mathbf{d} - \mathcal{L}^{th}\dot{\vartheta} - [(\mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}) : \mathbf{P}]\frac{1}{T_m}\left\langle \Phi\left(\frac{f}{\kappa} - 1\right)^m\right\rangle, \\ \dot{\vartheta} &= -\frac{1}{\rho c_p}\operatorname{div}\mathbf{q} + \frac{\vartheta}{c_p \rho_{\text{Ref}}}\frac{\partial \boldsymbol{\tau}}{\partial \vartheta} : \mathbf{d} + \frac{\chi}{\rho c_p}\boldsymbol{\tau} : \mathbf{d}^p, \end{aligned} \tag{9}$$

where L_υ defines the Lie derivative with respect to the velocity field, dot denotes the material derivative and ρ is actual density,

$$\mathcal{L}^e = \rho_{\text{Ref}}\frac{\partial^2 \hat{\psi}}{\partial \mathbf{e}^2}, \qquad \mathcal{L}^{th} = -\rho_{\text{Ref}}\frac{\partial^2 \hat{\psi}}{\partial \mathbf{e}\partial \vartheta}, \qquad c_p = -\vartheta\frac{\partial^2 \hat{\psi}}{\partial \vartheta^2}, \tag{10}$$

χ is the irreversibility coefficient.

To make possible numerical investigation of the three-dimensional dynamic adiabatic deformations of a body for different ranges of strain rate we introduce some simplifications of the constitutive model. The infinitesimal linear theory of elasticity is postulated with G and K as the shear and bulk modulus, respectively.

3. Formulation of an adiabatic inelastic flow process

To formulate the system of governing equations one has to complete formulae (9) with those that describe equilibrium and evolution of mass density.

Then, defining an adiabatic inelastic flow process [23, 24], one can simplify the system to the form:

Find ϕ, $\boldsymbol{\upsilon}$, ρ, $\boldsymbol{\tau}$ and ϑ as function of t and $\mathbf{x}$ such that

1. the field equations

$$\dot{\boldsymbol{\varphi}} = \mathcal{A}(t, \mathbf{x}, \boldsymbol{\varphi})\boldsymbol{\varphi} + \mathbf{f}(t, \mathbf{x}, \boldsymbol{\varphi}), \tag{11}$$

where

$$\boldsymbol{\varphi} = \begin{bmatrix} \phi \\ \boldsymbol{\upsilon} \\ \rho \\ \boldsymbol{\tau} \\ \vartheta \end{bmatrix}, \qquad \mathbf{f} = \begin{bmatrix} \boldsymbol{\upsilon} \\ 0 \\ 0 \\ -\left[\left(\mathcal{L}^{th}\frac{\chi}{\rho c_p}\boldsymbol{\tau} + \mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}\right) : \mathbf{P}\right]\frac{1}{T_m}\left\langle \Phi\left(\frac{f}{\kappa} - 1\right)\right\rangle \\ \frac{\chi}{\rho c_p}\boldsymbol{\tau} : \mathbf{P}\frac{1}{T_m}\left\langle \Phi\left(\frac{f}{\kappa} - 1\right)\right\rangle \end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\boldsymbol{\tau}}{\rho_{\mathrm{Ref}}\rho}\,\mathrm{grad} & \frac{1}{\rho_{\mathrm{Ref}}}\,\mathrm{div} & 0 \\ 0 & -\rho\,\mathrm{div} & 0 & 0 & 0 \\ 0 & \mathcal{L}^e : \mathrm{sym}\frac{\partial}{\partial \mathbf{x}} + 2\mathrm{sym}\left(\boldsymbol{\tau} : \frac{\partial}{\partial \mathbf{x}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

2. the boundary conditions
   (a) displacement ϕ is prescribed on a part ∂_ϕ of $\partial\phi(\mathcal{B})$ and tractions $(\boldsymbol{\tau}\cdot\mathbf{n})^a$ are prescribed on part ∂_τ of $\partial\phi(\mathcal{B})$, where $\partial_\phi \cap \partial_\tau = 0$ and $\partial_\phi \cup \partial_\tau = \partial\phi(\mathcal{B})$;
   (b) heat flux $\mathbf{q}\cdot\mathbf{n} = 0$ is prescribed on $\partial\phi(\mathcal{B})$;
3. the initial conditions

   ϕ, $\boldsymbol{\upsilon}$, ρ, $\boldsymbol{\tau}$ and ϑ are given at each particle $X \in \mathcal{B}$ at $t = 0$;

will be satisfied.

In Eq. $(12)_3$ $\frac{\partial \boldsymbol{\upsilon}}{\partial \mathbf{x}}$ defines the spatial velocity gradient and ρ_{Ref} denotes density in the reference configuration.

4. Wave propagation and dispersion

The imposed loading or boundary conditions controlled by displacements and velocities transfer the signal from one part of the structure to the other and distribute the energy. The signal propagates with velocity recognized by a quantity determined at any time and space location during dynamic process that is under consideration. To observe a propagation of a wave in the structure we focussed on the analysis of material points velocities and their accelerations. For classical plasticity the system of governing equations is hyperbolic. The change of the type of partial differential equations that could be the effect of softening becomes the limit of wave propagation. The further description of the process after initiation of localization is problematic and no unique solution can be obtained any more. The situation changes when viscosity is introduced. Dispersive waves are propagated during the dynamic process. Wave propagation is restricted by strain localization but the solution of the system of governing equations do not present substantial difficulties. Unique numerical solutions are obtained and the computations can be continued up to advanced stages of structure deformations.

The aim of the study is to demonstrate the existence of wave propagation effects within the problems of structure failure. Research is provided to check the relation between wave propagation and strain localization and the influence of parameters specifying the physical model and initial and boundary conditions. Different placement of localization is evaluated for various boundary velocities. Wave interaction influences the energy distribution, increase of stresses and finally the form of deformation. The plastic strain localization and in consequence the later place of failure is strictly related to wave evolution. The wave history makes possible to answer the question where the localization will take place. Waves development with their interference are studied as a reason of specific placement of localization zones.

In point of fact the dispersion of a waveform is caused by certain physical and/or geometrical characteristics of the medium in which the wave is generated. Consequently, instead of dispersive waves, it is perhaps more precise to speak of *a dispersive medium* or, where geometrical features alone cause the dispersion, *a dispersive geometry*, cf. Thau [32].

The relaxation time T_m (or viscosity) can be viewed either as a regularization parameter or as a microstructural parameter to be determined from experimental observations.

It has been proven that the localization of plastic deformation phenomenon in an elastic-viscoplastic solid body can arise only as the result of the reflection and interaction of waves. It has different character than that which occurs in a rate independent elasto-plastic solid body [22, 24]. Rate dependency (viscosity) allows the spatial difference operator in the governing equations to retain its ellipticity and the initial value problem is well-posed. Viscosity introduces implicitly a length-scale parameter into the dynamical initial-boundary value problem and hence it implies that the localization region is diffused when compared with an inviscid plastic material. In the dynamical initial-boundary value problem the stress and deformation due to wave reflections and interactions are not uniformly distributed, and this kind of heterogeneity can lead to strain localization in the absence of geometrical or material irregularities. This kind of phenomenon has been noticed by Nemes and Eftis [17] (cf. also the results by Sluys et al. [30]).

The theory of viscoplasticity gives the possibility to obtain mesh-insensitive results in localization problems with respect to the width of the shear band and the wave reflection and interaction patterns (cf. Sluys et al. [30]).

From the point of view of our present study the most important feature is that the propagation of deformation waves in an elastic-viscoplastic medium has dispersive nature. In this study we shall use the numerical finite element procedure to show the solution of the particular evolution problems with nonlinear dissipative and dispersive wave effects. For this purpose the set of numerical studies with varying relaxation times is presented. The total dissipated energy for rate independent material is only that which is consumed by plastic deformations; however, for rate dependent (viscoplastic) material the part of dissipative energy which comes from the dispersive character of waves additionally appears.

The application of the variational method and the perturbation theory in the investigation of the nonlinear evolution problems will be presented by the authors in the forthcoming papers.

The numerical solutions of the initial-boundary value problem (evolution problem) were discussed in [6]. Mathematical formulation of the evolution problem was presented. Discretisations in space and time were there proposed and convergence, consistency and stability were examined. The Lax–Richtmayer equivalence theorem was also formulated and conditions under which this theorem is valid were investigated as well.

5. Numerical examples and results

The thin rectangular plate is the subject of our interest. The numerical study of localized adiabatic shear band in inelastic solids is presented for variable loading conditions. The examples are presented for a thin steel plate specimens subjected to tension, with the controlled displacements imposed at one side with different velocities. Basically, asymmetric constraints of the specimen are considered (velocities act at one side, while the opposite is fixed). It leads to non-symmetric final deformations. The examples were computed for different initial velocities as well as different relaxation times. In the numerical examples we studied the sensitivity of plastic zones location to the variable loads and constitutive parameters. The distributions of plastic equivalent strains PEEQ and temperature, vector plots of particle velocities represent the results. The computations were performed using the environment of finite element program ABAQUS/Explicit. Numerical data were compared with the experimental results.

There was tested a rectangular strip plate of the length $l = 25.4$ mm, the width $w = 12.7$ mm. The adiabatic case of this thin plate ($t = 0.33$ mm) was modeled using 4-node shell elements. The following data were accepted as: Young modulus $E = 200,000$ MPa, yield limit $\sigma_P = 1634$ MPa, mass density $\rho = 7850$ kg/m^3. The relaxation time of the material $T_m = 2.5 \cdot 10^{-6}$ s, specific heat 460 J/kg K and heat fraction 0.9. The computed set of examples includes one-sided symmetrically loaded specimens.

In Fig. 1 the deformed meshes and the qualitative distribution of plastic equivalent strains for different velocities are presented. The velocities applied to the top edge varies between 2 m/s up to 50 m/s (2, 5, 10, 20, 50 m/s) and the plotted results are monitored for the same elongation of the specimens. The small geometric imperfection (statical horizontal movement of top side) has been assumed. One can observe that for relatively slow processes the mode of deformation is not symmetric (usually observed in quasi-static experiments) while for higher velocities despite of initial imperfections always become symmetric. This kind of behaviour is well known and observable in dynamic laboratory tests.

Figure 2 presents the results of sensitivity of the localization zone width to the accepted constitutive parameter T_m for fixed initial velocity 20 m/s. The relaxation time changes between $2.5 \cdot 10^{-2}$ s up to $2.5 \cdot 10^{-6}$ s.

As it is expected the values of the forces which accompany these states differ reaching the higher limit values for faster processes. The softening character of the specimens' behaviour is clearly visible when looking at the generalized results (e.g. in the spaces of acting forces and displacements).

The important observation confirm that the place of localization is chosen only as an effect of waves interaction and significantly depends on the initial conditions.

It is clearly visible that for different velocities (not quasi-static) the places of strain localization appear symmetrically but with different intensity.

The level of diffusion of the zone of localization significantly depends on the constitutive parameter, namely the relaxation time T_m. For shorter T_m the width of localized zone is smaller.

Figure 3 presents the vector plots of velocities of particles in the specimen for different stages of the process. The assumed initial velocity $v = 20$ m/s and the relaxation time $T_m = 2.5 \cdot 10^{-6}$ s. At the beginning of the process the longitudinal waves propagate with the elastic speed. Also from the very beginning the refraction of waves at the edges is clearly visible. The vector plot (arrows) shows the directions of the particle movements and the length of arrows reflect their values. In Fig. 3 the crucial role of reflection of waves and its interaction is observed. For 1/10 of the process the place of plastic strain localization is fixed. In the vicinity of the place of localization the speed of waves are almost equal to zero. In the areas where the localization appears the velocities and also the accelerations are close to zero. For the initial velocity of 20 m/s a set of results with different relaxation times $T_m = 2.5 \cdot 10^{-5}$; $2.5 \cdot 10^{-6}$; $2.5 \cdot 10^{-7}$ is presented, for the fine meshes (80×40). In Fig. 4 the values of the total energy divided into elastic and dissipated parts are plotted. For this viscoplastic media, if T_m tends to 0 at the limit case the material becomes rate independent, so the dissipated energy is only the part that is consumed for plastic deformations. The differences

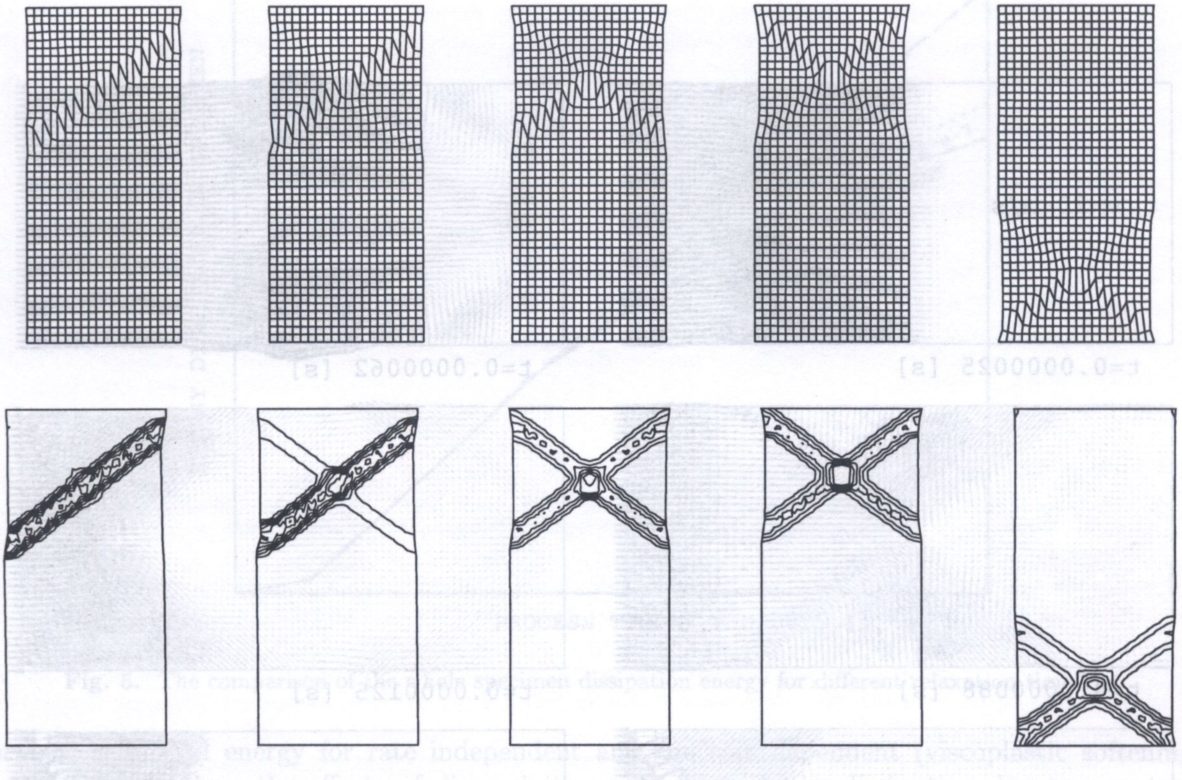

Fig. 1. Deformed meshes (top) and places of strain localization (bottom) for different loading velocities

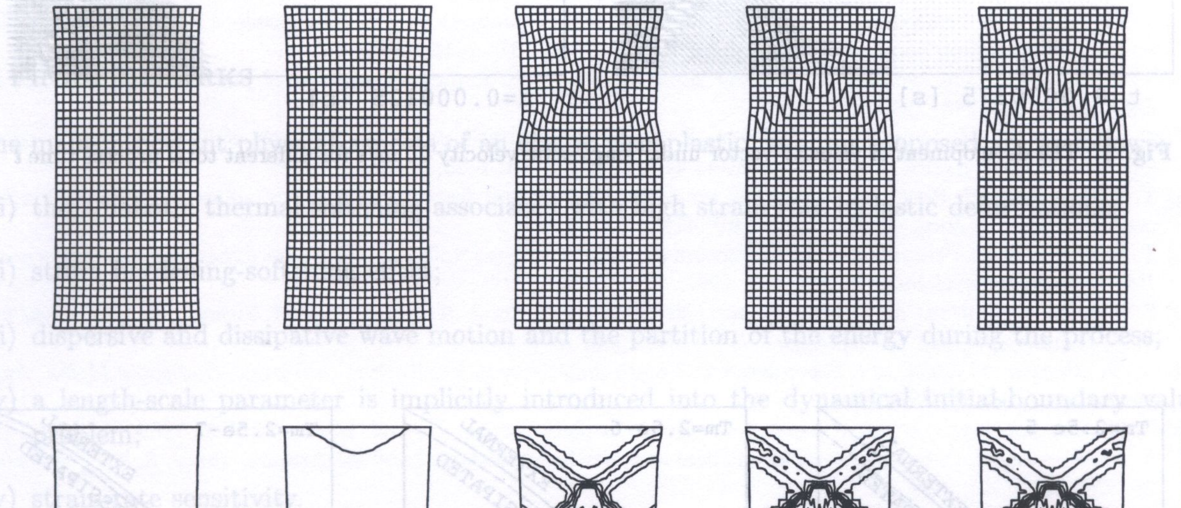

Fig. 2. Deformed meshes (top) and places of strain localization (bottom) for different relaxation times

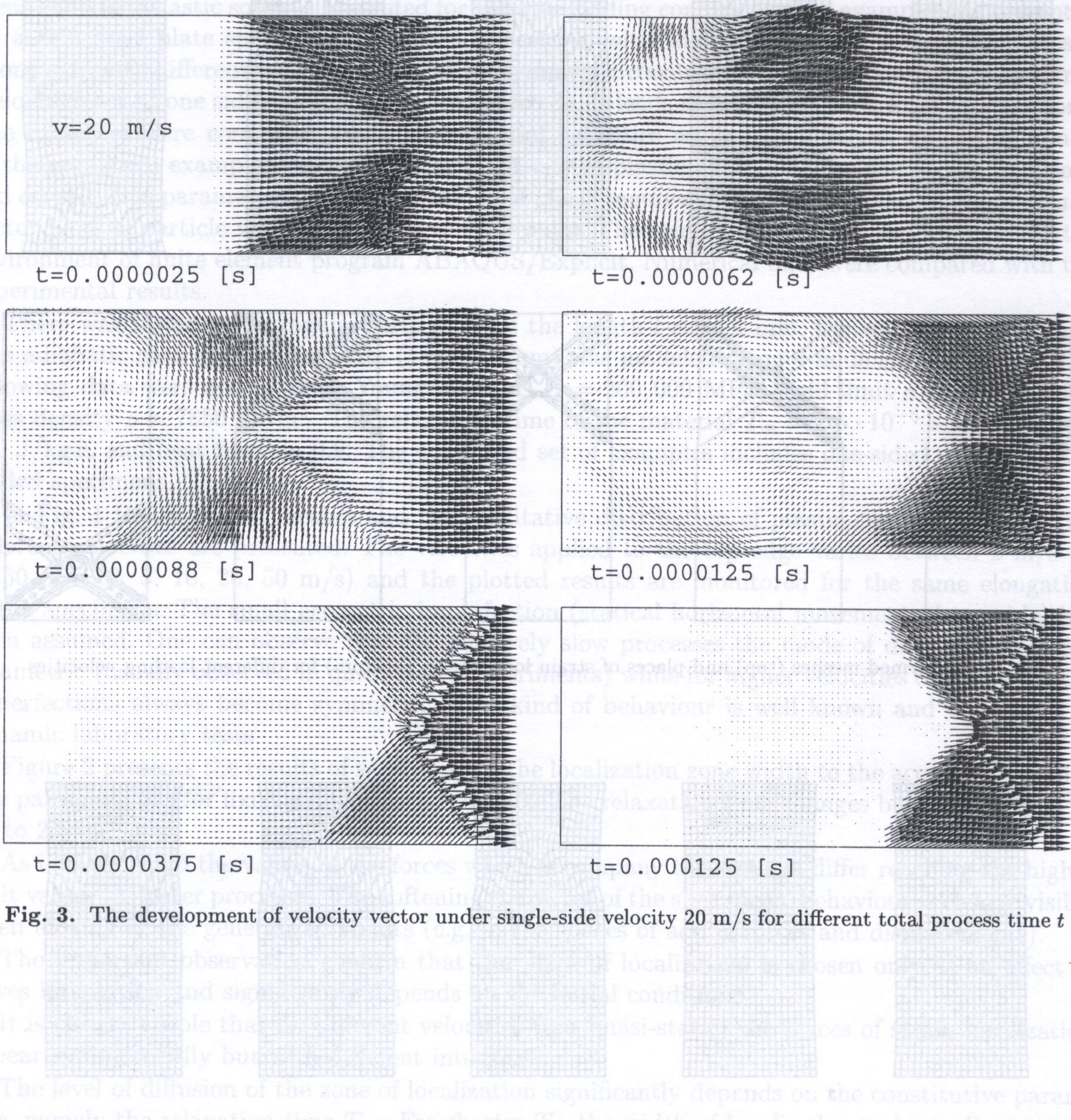

Fig. 3. The development of velocity vector under single-side velocity 20 m/s for different total process time t

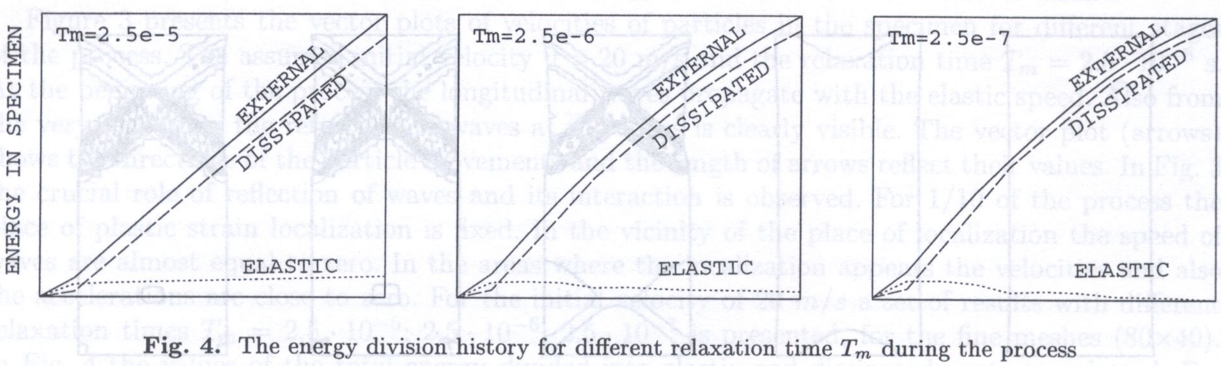

Fig. 4. The energy division history for different relaxation time T_m during the process

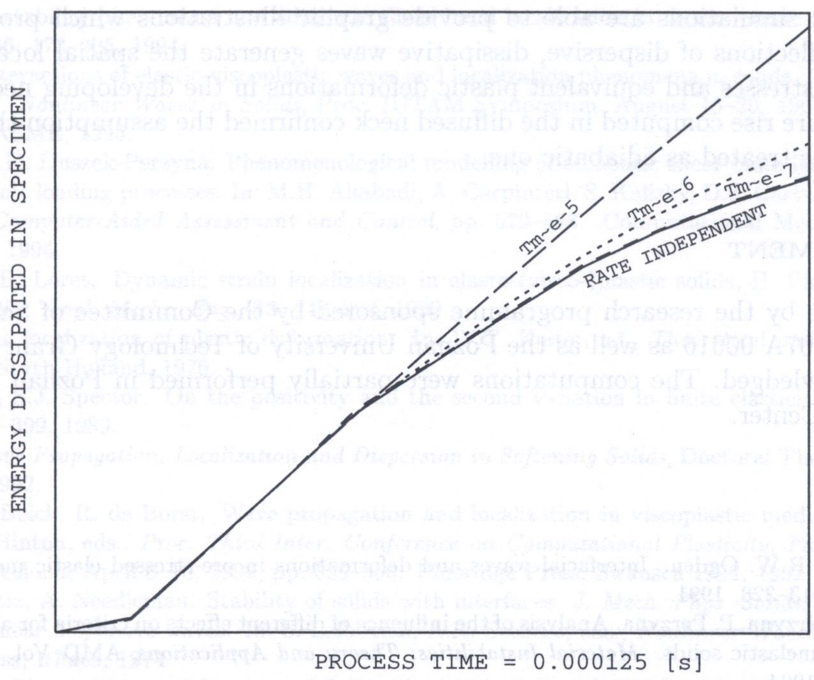

Fig. 5. The comparison of the whole specimen dissipation energy for different relaxation time T_m

between dissipated energy for rate independent and the rate dependent (viscoplastic softening) models (Fig. 5) show the effects of dispersivity combined with the dissipation due to overstress states. There is a certain amount of energy which has to be used for regularization of the initial boundary value problem. For very short relaxation times the stability of the solution can not be assured.

6. Final remarks

The main important physical aspects of an elastic-viscoplastic medium proposed are as follows:

(i) the adiabatic thermal softening associated with high strain rate inelastic deformations;

(ii) strain hardening-softening effect;

(iii) dispersive and dissipative wave motion and the partition of the energy during the process;

(iv) a length-scale parameter is implicitly introduced into the dynamical initial-boundary value problem;

(v) strain-rate sensitivity.

The numerical simulations of the evolution problems have shown that for dynamically imposed loading the inhomogeneous fields of stress and deformation are caused by the propagation of waves and the reflection of waves. This interaction of waves induced necking at different locations along the specimen depending upon the strain rate imposed. The phenomenon of localization of plastic deformation occurs without imposition of any geometrical, thermal or material imperfections. In each of the numerical examples considered due to different initial conditions the evolution of the necking looks differently. Propagative waves have dispersive and dissipative nature and this fact has fundamental influence on the development of localization of plastic deformation in a mode of necking.

The numerical simulations are able to provide graphic illustrations which proved that the interactions and reflections of dispersive, dissipative waves generate the spatial localization and the intensification of stresses and equivalent plastic deformations in the developing neck.

The temperature rise computed in the diffused neck confirmed the assumption that the evolution process have to be treated as adiabatic one.

Acknowledgment

Financial support by the research programme sponsored by the Committee of Scientific Research under Grant 7 T 07A 00616 as well as the Poznań University of Technology Grant DS-11-024/2001 are kindly acknowledged. The computations were partially performed in Poznan Supercomputing and Networking Center.

References

[1] M.A. Dowaikh, R.W. Ogden. Interfacial waves and deformations in pre-stressed elastic media. *Proc. R. Soc. Lond.*, **A433**: 313–328, 1991.

[2] M.K. Duszek-Perzyna, P. Perzyna. Analysis of the influence of different effects on criteria for adiabatic shear band localization in inelastic solids. *Material Instabilities: Theory and Applications*, AMD–Vol. 183 / MD–Vol. 50, ASME, 59–85, 1994.

[3] M.K. Duszek-Perzyna, P. Perzyna. Acceleration waves in analysis of adiabatic shear band localization. In: J.L. Wegner, F.R. Norwood, eds., *Nonlinear Waves in Solids*, Proc. IUTAM Symposium, August 15–20, 1993, Victoria, Canada, pp. 128–135. ASME, 1995.

[4] M.K. Duszek-Perzyna, P. Perzyna. Analysis of anisotropy and plastic spin effects on localization phenomena. *Archive of Applied Mechanics*, **68**: 352–374, 1998.

[5] A. Glema, W. Kąkol, T. Łodygowski. Numerical modelling of adiabatic shear band formation in a twisting test. *Engineering Transactions*, **45**(3-4): 419–431, 1997.

[6] A. Glema, T. Łodygowski, P. Perzyna. Interaction of deformation waves and localization phenomena in inelastic solids. *Int. Journ. Comp. Appl. Mech. Eng.*, **183**: 123–140, 2000.

[7] J. Hadamard. *Lecons sur la Propagation des Ondes et les Equations de l'Hydrodynamique*, Chap. 6. Paris, 1903.

[8] R. Hill. Acceleration wave in solids. *J. Mech. Phys. Solids*, **10**: 1–16, 1962.

[9] T. Łodygowski. On avoiding of spurious mesh sensitivity in numerical analysis of plastic strain localization. *CAMES*, **2**: 231–248, 1995.

[10] T. Łodygowski, M. Lengnick, P. Perzyna, E. Stein. Viscoplastic numerical analysis of dynamic plastic strain localization for a ductile material. *Archives of Mechanics*, **46**: 1–25, 1994.

[11] T. Łodygowski, P. Perzyna. Numerical modelling of localized fracture of inelastic solids in dynamic loading processes. *Int. J. Num. Meth. Engng.*, **40**: 4137–4158, 1997.

[12] T. Łodygowski, P. Perzyna. Localized fracture in inelastic polycrystalline solids under dynamic loading processes. *Int. J. Damage Mechanics*, **6**: 364–407, 1997.

[13] J. Mandel. Conditions de stabilité et postulat de Drucker. In: J. Kravchenko, P.M. Sirieys, eds., *Rheology and Soil Mechanics*, pp. 58–68. Springer, New York, 1966.

[14] A. Needleman. Material rate dependence and mesh sensitivity in localization problems. *Computer Meths. Appl. Mechs. Eng.*, **67**: 69–85, 1988.

[15] A. Needleman. Dynamic shear band development in plane strain. *J. Appl. Mech.*, **56**: 1–9, 1989.

[16] A. Needleman, M. Ortiz. Effects of boundaries and interfaces on shear-band localization. *Int. J. Solids Structures*, **28**: 859–877, 1991.

[17] J.A. Nemes, J. Eftis. Constitutive modelling on the dynamic fracture of smooth tensile bars. *Int. J. Plasticity*, **9**: 243–270, 1993.

[18] R.W. Ogden. Nonlinear effects associated with waves in pre-stressed elastic solids near bifurcation points. In: J.L. Wegner, F.R. Norwood, eds., *Nonlinear Waves in Solids*, Proc. IUTAM Symposium, August 15–20, 1993, Victoria, Canada, pp. 109–113. ASME, 1995.

[19] N.S. Ottosen, K. Runesson. Acceleration waves in elasto-plasticity. *Int. J. Solids Structures*, **28**: 135–159, 1991.

[20] P. Perzyna. The constitutive equations for rate sensitive plastic materials. *Quart. Appl. Math.*, **20**: 321–332, 1963.

[21] P. Perzyna. Thermodynamic theory of viscoplasticity. *Advances in Applied Mechanics*, **11**: 313–354, 1971.

[22] P. Perzyna. Adiabatic shear band localization fracture of solids in dynamic loading processes. In J. Harding, ed., *Proc. of Int. Conference on Mechanical and Physical Behaviour of Materials under Dynamic Loading*, Oxford, September 26–30, 1994, pp. 441–446. Les Editions de Physique Le Ulis 1994, 1994.

[23] P. Perzyna. Instability phenomena and adiabatic shear band localization in thermoplastic flow processes. *Acta Mechanica*, **106**: 173–205, 1994.

[24] P. Perzyna. Interactions of elastic-viscoplastic waves and localization phenomena in solids. In: J.L. Wegner, F.R. Norwood, eds., *Nonlinear Waves in Solids*, Proc. IUTAM Symposium, August 15–20, 1993, Victoria, Canada, pp. 114–121. ASME, 1995.

[25] P. Perzyna, M.K. Duszek-Perzyna. Phenomenological modelling of adiabatic shear band localization fracture of solids in dynamic loading processes. In: M.H. Aliabadi, A. Carpinteri, S. Kalisky, D.J. Cartwright, eds., *Localized Damage III; Computer-Aided Assessment and Control*, pp. 579–588. Computational Mechanics Publications, Southampton, 1994.

[26] J.H. Prevost, B. Loret. Dynamic strain localization in elasto-(visco-)plastic solids, II. Plane strain examples. *Computer Meths. Appl. Mechs. Eng.*, **83**: 247–294, 1990.

[27] J.R. Rice. The localization of plastic deformation. In: W.T. Koiter, ed., *Theoretical and Applied Mechanics*, pp. 207–220. North-Holland, 1976.

[28] H.C. Simpson, S.J. Spector. On the positivity and the second variation in finite elasticity. *Arch. Rat. Mech. Anal.*, **32**: 369–399, 1989.

[29] L.J. Sluys. *Wave Propagation, Localization and Dispersion in Softening Solids*, Doctoral Thesis. Delft University Press, Delft, 1992.

[30] L.J. Sluys, J. Bolck, R. de Borst. Wave propagation and localization in viscoplastic media. In: D.R.J. Owen, E. Oñate, E. Hinton, eds., *Proc. Third Inter. Conference on Computational Plasticity, Fundamentals and Applications*, Barcelona, April 6–10, 1992, pp. 539–550. Pineridge Press, Swansea 1992, 1992.

[31] Z. Suo, M. Ortiz, A. Needleman. Stability of solids with interfaces. *J. Mech. Phys. Solids*, **40**: 613–640, 1992.

[32] S.A. Thau. Linear dispersive waves. In: S. Leibovich, A.R. Seebass, eds., *Nonlinear Waves*, pp. 44–81. Cornell University Press, Ithaca, 1974.

[33] T.Y. Thomas. *Plastic Flow and Fracture of Solids.* Academic Press, New York, 1961.

[34] C. Truesdell. General and exact theory of waves in finite elastic strain. *Arch. Rat. Mech. Anal.*, **19**: 263–296, 1965.

[35] W.M. Wang. *Stationary and Propagative Instabilities in Metals – A Computational Point of View*, Doctoral Thesis. Delft University Press, Delft, 1997.

[36] W.M. Wang, L.J. Sluys, R. de Borst. Interaction between material length scale and imperfection size for localization phenomena in viscoplastic media. *European J. Mechanics*, **15**: 447–464, 1996.

[37] G.B. Whitham. *Linear and Nonlinear Waves.* John Wiley, New York, 1974.