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Application of cellular automata simulation to truss structure design

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In the cellular automata simulation, the object under consideration is divided into small cells and the simulation is performed according to the local rule which is defined as the local relationship among cells. In this paper, the concept of cellular automata is applied to the design scheme of truss structures. First, truss elements are considered as the cells of the cellular automata and the local rule is derived from the optimization problem. The objective functions are defined to minimize the total weight of the structure and to obtain even stress distribution in the whole structure. The constraint conditions are introduced in order to define the local rule.

The present method is applied to the design of the plane and the three-dimensional truss structures such as Schwedler and Lamella Domes. The convergence histories of the total weight and the mean and the maximum stresses are shown in order to discuss the property of the present method.

Keywords: structural design, cellular automata (CA), local rule, truss structure

1. Introduction

In cellular automata simulations, the object under consideration is uniformly divided into small cells and states of each cell are updated according to so-called "local rules". Since local rules are defined as local relationships among neighboring cells, the simulation schemes are very effective for the phenomena of which global governing equations cannot be defined clearly. Therefore, many researchers have studied the application of the cellular automata to phenomena such as pattern analysis of butterfly and spiral shell, traffic and pedestrian flow and so on [1–6]. On the other hand, the application of cellular automata simulation to truss structure design is described in this paper.

Some researchers have presented structural design schemes based on the concept of the cellular automata [7–29]. Xie et al. have presented Evolutionary Structural Optimization (ESO) method in 1993 [7–21]. We recognize that Evolutionary Structure Optimization method is the first application of cellular automata simulation to structural designs. In the method, a design domain is uniformly divided into small elements or cells and finite element analysis is performed for whole structure. Some design parameters named as rejection ratio (RR), evolutionary ratio (ER) and so on are defined and then, removal and addition of cells are performed according to the relationship among the design parameters and the physical quantities at each cell such as stress.

Definition of local rules is very important in the cellular automata simulation. Local rules in existing studies can be classified into experimental, evolutionary, biomechanical and mathematical formulations. In the experimental formulation, the local rules are defined as nonlinear functions among stress states and design variables [22, 23]. Since the nonlinear functions are determined from numerical experiments, the derived rules depend on the problems to be solved. In the evolutionary formulation, the local rules are defined by help of genetic algorithm, artificial neural network or L-system [24–26, 30]. Also in these cases, the rules should be learned by the help of the experimental data and so on. By the way, a remodeling equation of bones shows that the bone density depends on the strain energy and the volume of the load-applied bones. In the biomechanical formulation, the local rule is derived from the remodeling equation [27, 28].

On the other hand, the mathematical formulation, which is employed in this paper, derives the local rule from the optimization problem. The global optimization problem for the whole structure is defined from two objective functions and then, the introduction of the special constraint conditions transforms the global optimization problem for the whole structure to the local optimization problem for the local structure. The local rule is derived from the local optimization problem defined from two objective functions and the constraint conditions. This formulation is based on the so-called gradient-type optimization procedure from the theoretical viewpoints. However, it is not necessary to estimate the gradients of the objective functions (the design sensitivities) actually. This is because the design sensitivities are derived as the local rule between the neighboring cells. The rule has been successfully applied to continuum structural design in the previous study [31]. The rule is developed to the design of truss structures in this paper.

Truss structures, which consist of bar elements, are widely used for large-scaled structures such as bridges, towers, space stations and so on. The optimization of the truss structures have been already presented by some researchers [32–36]. Especially, the ground structure method presented in the reference [32] have given the important solutions from the practical applicability. When applying the concept of the cellular automata to the truss structure design, there is some difficulties to be overcome. The first problem is to represent the truss structures by cells. The truss structures consist of bar elements and therefore, the elements are considered as cells of cellular automata to define a local rule. The rule is applied to design of plane truss and three-dimensional dome structures.

This paper is organized as follows. In Sec. 2, the stress analysis in the truss structure is explained briefly for convenience of the derivation of the explanation for local rules in the next section. In Sec. 3, a concept of cells is developed to express truss structures by cells and to define cell relationship. Then, a local rule is derived from an optimization problem consisted of two objective functions and constraint conditions and then, an algorithm of the present scheme is described. In Sec. 4, the present scheme is applied to design of plane truss and three-dimensional dome structures. The discussions are summarized in Sec. 5.

2. Stress analysis of truss structure

We shall explain briefly the stress analysis in the truss structures for convenience of derivation of the explanation for local rules [37, 38].

An element of a truss structure is shown in Fig. 1. The parameter θ denotes the angle from x-coordinate to the element and the nodes on both ends of the element are as 1 and 2, respectively. We shall denote the displacement components in x- and y-directions at node i ($i = 1, 2$) as u_{i1} and u_{i2} and external forces at a node i as f_{i1} and f_{i2}, respectively.

A stiffness equation can be derived from finite element formulation as follows.

$$\mathbf{K}_e \mathbf{u}_e = \mathbf{f}_e, \tag{1}$$

where $\mathbf{f}_e$, $\mathbf{u}_e$ and $\mathbf{K}_e$ denote the external force and displacement vectors and the stiffness matrix at the element, respectively, which are defined as

$$\mathbf{u}_e = \{u_{11}, u_{12}, u_{21}, u_{22}\}^{\mathrm{T}}, \tag{2}$$

$$\mathbf{f}_e = \{f_{11}, f_{12}, f_{21}, f_{22}\}^{\mathrm{T}}, \tag{3}$$

$$\mathbf{K}_e = \frac{EA}{L}\widetilde{\mathbf{K}}_e, \tag{4}$$

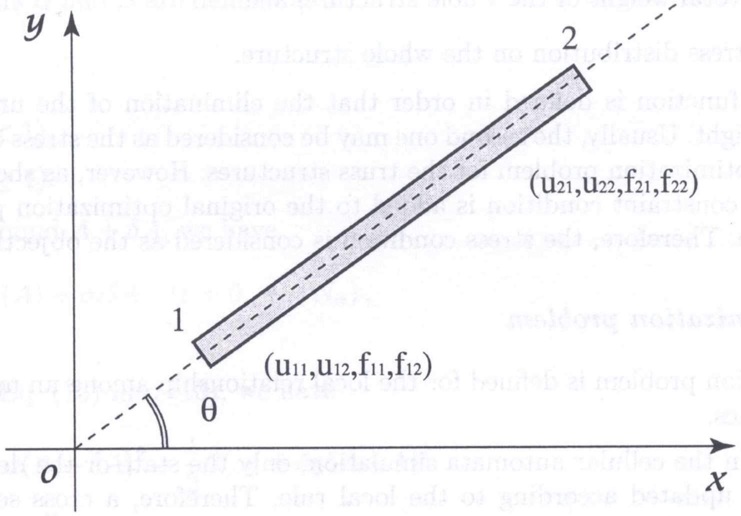


Fig. 1. Truss element

where E, A and L denote the Young's modulus, the cross-sectional area and the length of the element, respectively.

Holding Eq. (1) at each element of a structure and arranging them in matrix form, we have

$$\mathbf{K}\mathbf{u} = \mathbf{f}, \tag{5}$$

where $\mathbf{f}$, $\mathbf{u}$ and $\mathbf{K}$ denote global external force and displacement vectors and global stiffness matrix, respectively.

A stress at a element σ is estimated from the displacements:

$$\sigma = \mathbf{B}_e \mathbf{u}_e\,, \tag{6}$$

where $\mathbf{B}_e$ denotes the vector related to the stress-strain relationship.

3. Derivation of local rule

3.1. Cell-representation of truss structure

In the cellular automata simulation, the object domains are usually divided into small cells. Since truss structures are composed of bar elements, it is difficult to express them by cells of cellular automata. To avoid this problem, bar elements are considered as cells in cellular automata simulation to define a local rule. All elements connecting to an updated element are basically accounted for neighborhood elements for the element. The updated element is numbered as 0 and the other numbered elements are neighborhood elements for the element-0.

3.2. Optimization problem

3.2.1. Global optimization problem

In the present method, the global optimization problem for the whole structure is transformed into the local optimization problem by using the constraint conditions.

As the objective functions of the global optimization problem, we shall take

1) to minimize the total weight of the whole structure, and

2) to obtain even stress distribution on the whole structure.

The first objective function is defined in order that the elimination of the unnecessary elements reduces the total weight. Usually, the second one may be considered as the stress constraint condition in the traditional optimization problem for the truss structures. However, as shown in the following section, the special constraint condition is added to the original optimization problem in order to derive the local rule. Therefore, the stress condition is considered as the objective function.

3.2.2. Local optimization problem

The local optimization problem is defined for the local relationship among an updated element and its neighborhood ones.

Design variable. In the cellular automata simulation, only the state or the design variable of the updated element is updated according to the local rule. Therefore, a cross sectional area of the updated element A is considered as the design variable of the local optimization problem.

Objective functions. In the global optimization problem, two design objectives are considered; minimization of the total weight of the whole structure, and obtaining even stress distribution on the whole structure.

In the local optimization problem, the objective function for minimizing the total weight can be defined by the weight of the updated element as follows.

$$H_1 = \frac{1}{2}\left(\frac{\tilde{A}}{A_0}\right)^2 \equiv \frac{1}{2}A^2, \tag{7}$$

where $\tilde{A}$ and A_0 denote cross sectional area of the element and its initial value, respectively.

The objective function for obtaining even stress distribution can be defined from the deviation between the stress and the reference stress as follows.

$$H_2 = \frac{1}{2}\left(\frac{|\tilde{\sigma}_0|}{\sigma_c} - 1\right)^2 \equiv \frac{1}{2}(\sigma_0 - 1)^2, \tag{8}$$

where $\tilde{\sigma}_0$ and σ_c denote the stress at the element and the yield stress of the material, respectively.

Constraint condition. If the area of the updated element is changed independently, the equilibrium state between the element and the neighborhood ones may b broken. The constraint condition is defined so as to insure that the stress states at neighborhood elements are almost insensitive with respect to the variation of the design variable.

$$g_i = \frac{\tilde{\sigma}_i}{\tilde{\sigma}_i^0} - 1 \equiv \sigma_i - 1 = 0 \quad (i = 1, \cdots, N_n), \tag{9}$$

where $\tilde{\sigma}_i$ and $\tilde{\sigma}_i^0$ denote stresses at a neighborhood element i at present and previous iteration steps, respectively. N_n denotes total number of neighborhood elements.

3.3. Penalty function

By introducing penalty parameter p and weight parameters α and β, we have the penalty function

$$\begin{aligned} H &= \alpha H_1 + \beta H_2 + \frac{p}{2}\sum_{i=1}^{N_n} g_i^2 \\ &= \alpha\frac{1}{2}A^2 + \frac{1}{2}\beta(\sigma_0 - 1)^2 + \frac{p}{2}\sum_{i=1}^{N_n}(\sigma_i - 1)^2, \end{aligned} \tag{10}$$

where the parameters α and β are defined as

$$\alpha+\beta=1 \tag{11}$$

$$\beta=\begin{cases}\sigma_0 & (\sigma_0<1),\\ 1 & (\sigma_0\geq 1).\end{cases} \tag{12}$$

Expanding σ_i around $A+\delta A$, we have

$$\sigma_i(A+\delta A)\simeq\sigma_i(A)+\acute{\sigma}_i\delta A \quad (i=0,\cdots,N_n)\,, \tag{13}$$

where $(\acute{\ }) = \partial/\partial A$.

By substituting Eq. (13) into (10), we have

$$H(A+\delta A)\simeq\frac{1}{2}\alpha(A+\delta A)^2+\frac{1}{2}\beta(\sigma_0+\acute{\sigma}_0\delta A-1)^2$$
$$+\frac{p}{2}\sum_{i=1}^{N_n}(\sigma_i+\acute{\sigma}_i\delta A-1)^2. \tag{14}$$

Minimizing Eq. (14) with respect to δA, we have

$$\frac{\partial H(A+\delta A)}{\partial(\delta A)}=\alpha(A+\delta A)+\beta(\sigma_0+\acute{\sigma}_0\delta A-1)\acute{\sigma}_0$$
$$+p\sum_{i=1}^{N_n}(\sigma_i+\acute{\sigma}_i\delta A-1)\acute{\sigma}_i=0 \tag{15}$$

and rearranging, we have

$$\delta A=-\frac{\alpha A+\beta(\sigma_0-1)\acute{\sigma}_0+p\sum_{i=1}^{N_n}(\sigma_i-1)\acute{\sigma}_i}{\alpha+\beta\acute{\sigma}_0^2+p\sum_{i=1}^{N_n}\acute{\sigma}_i^2}\,. \tag{16}$$

Design sensitivities $\acute{\sigma}_0$ and $\acute{\sigma}_i$ in Eq. (9) are estimated as follows.

3.4. Approximate estimation of $\acute{\sigma}_0$

If the constraint condition is satisfied or almost satisfied, an updated element can be solved independently. A stiffness equation for the updated element is as follows.

$$\frac{EA}{L}\widetilde{\mathbf{K}}_e\mathbf{u}_e=\mathbf{f}_e. \tag{17}$$

Direct differentiation of this equation with respect to A leads to

$$\frac{E}{L}\widetilde{\mathbf{K}}_e\mathbf{u}_e+\frac{EA}{L}\widetilde{\mathbf{K}}_e\acute{\mathbf{u}}_e=\acute{\mathbf{f}}_e=\mathbf{0},$$
$$\acute{\mathbf{u}}_e=-\frac{L}{EA}(\widetilde{\mathbf{K}}_e)^{-1}\frac{E}{L}\widetilde{\mathbf{K}}_e\mathbf{u}_e=-\frac{\mathbf{u}_e}{A}\,. \tag{18}$$

Equation (18) denotes a displacement sensitivity.

A stress sensitivity is derived from direct differentiation of Eq. (6) with respect to A as follows.

$$\acute{\sigma}_0=\mathbf{B}_e\acute{\mathbf{u}}_e=-\mathbf{B}_e\frac{\mathbf{u}_e}{A}=-\frac{\sigma_0}{A}\,. \tag{19}$$

3.5. Approximate estimation of $\acute{\sigma}_i$

We shall consider the stiffness equation for a neighborhood element as follows.

$$\mathbf{K}_e\mathbf{u}_e = \mathbf{f}_e\,. \tag{20}$$

A right-hand side term $\mathbf{f}_e$ is a vector of external forces applied to the element. If a cross sectional area of the updated element A decreases, $\mathbf{f}_e$ increases. On the contrary, $\mathbf{f}_e$ possibly decreases if the cross sectional area A increases. Therefore, we shall assume that $\mathbf{f}_e$ are in inverse proportion to A;

$$A\mathbf{f}_e = \text{const.} \tag{21}$$

Direct differentiation of this equation with respect to A leads to

$$\begin{aligned} &\mathbf{f}_e + A\acute{\mathbf{f}}_e = 0,\\ &\acute{\mathbf{f}}_e = -\frac{\mathbf{f}_e}{A}\,. \end{aligned} \tag{22}$$

Besides, differentiating Eq. (20) with respect to A, we have

$$\mathbf{K}_e\acute{\mathbf{u}}_e = \acute{\mathbf{f}}_e\,. \tag{23}$$

Substituting Eq. (22) to (23), we have

$$\begin{aligned} \acute{\mathbf{u}}_e &= (\mathbf{K}_e)^{-1}\acute{\mathbf{f}}_e\\ &= -(\mathbf{K}_e)^{-1}\left(\frac{\mathbf{f}_e}{A}\right) = -\frac{\mathbf{u}_e}{A}\,. \end{aligned} \tag{24}$$

Direct differentiation of Eq. (6) with respect to A leads to

$$\acute{\sigma}_i = \mathbf{B}_e\acute{\mathbf{u}}_e = -\mathbf{B}_e\frac{\mathbf{u}_e}{A} = -\frac{\sigma_i}{A}\,. \tag{25}$$

3.6. Definition of local rule

Substituting Eqs. (19) and (25) to (16), we have

$$\Delta A = \frac{-\alpha A^2 + \beta(\sigma-1)\sigma + p\sum_{i=1}^{N_n}(\sigma_i - 1)\sigma_i}{\alpha A^2 + \beta\sigma^2 + p\sum_{i=1}^{N_n}\sigma_i^2}\cdot A\,. \tag{26}$$

The cross sectional area is updated with

$$A^{k+1} = A^k + \delta A\,, \tag{27}$$

where superscripts denote numbers of iteration steps.

3.7. Convergence criterion

A convergence criterion is defined from the convergence rate of the total weight of a structure as follows

$$\left|W^{k+1} - W^k\right| < \varepsilon\,, \tag{28}$$

where W and ε denote total weight of a structure and a positive number specified in advance. The superscript k is a number of iteration step.

3.8. Algorithm of present scheme

The algorithm of the present scheme is as follows.

1. Input initial data such as element data, boundary and design conditions.
2. Estimate stress distribution on the whole structure.
3. Check convergence criterion. If the criterion is satisfied, process goes to output results. If not so, the process goes to the next.
4. Calculate δA from Eq. (26) and update cross sectional area by Eq. (27).
5. Go to Step 2.

4. Numerical examples

4.1. Plane truss structure

A first example is a plane truss structure shown in Fig. 2. Design parameters are shown in Table 1. $\sigma^0_{\max}$ denotes a maximum stress at an initial assumed structure and initial cross sectional areas of each cell are $A^0 = 10^{-1}$ (m^2).

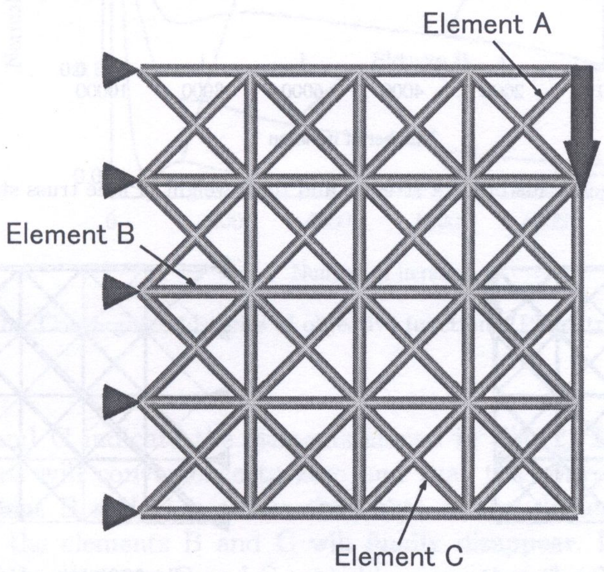


Fig. 2. Initial shape (Plane truss structure)

Table 1. Design parameters (Plane truss structure)

Number of elements	68
Young's modulus	$E = 1 \times 10^{10}$ Pa
Load	$P = 1000$ N
Initial cross section	$A^0 = 0.1$ m^2
Penalty coefficient	$p = 10$
Reference stress	$\sigma_c = \sigma^0_{\max}$

Convergence histories of stresses and a total weight of a structure are shown in Fig. 3. The abscissa and the left- and right-ordinates denote the number of iteration, stress divided by a reference stress

σ_c and a total weight of a structure W divided by its initial value W_0, respectively. The maximum stress $\sigma_{\max}$ increases slightly at primary iteration steps and converges to the reference stress at 5000th iteration step. The mean stress σ_{m} gradually decreases to 60% of the reference stress. The total weight of the structure W rapidly decreases and converges to 20% of the initial weight at 5000-th iteration step. Structure profiles at 1000, 2000, 5000 and 10000-th iteration steps are shown in Fig. 4. The results demonstrate that the structure converges to a truss structure consisted of two

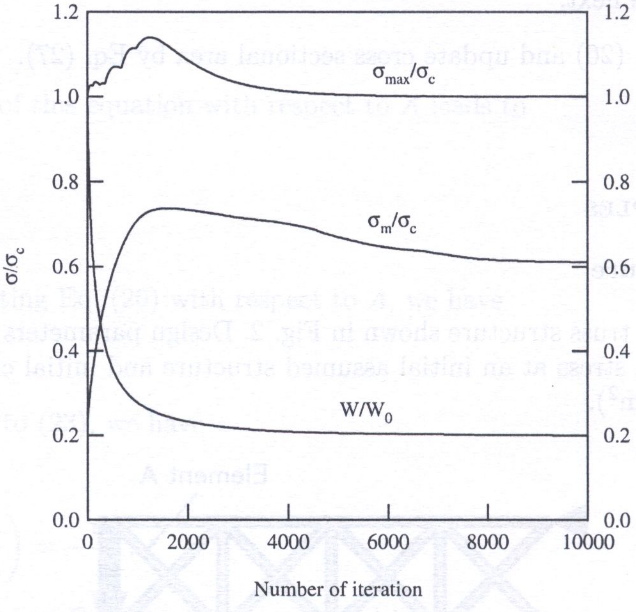


Fig. 3. Convergence histories of stresses and total weight (Plane truss structure)

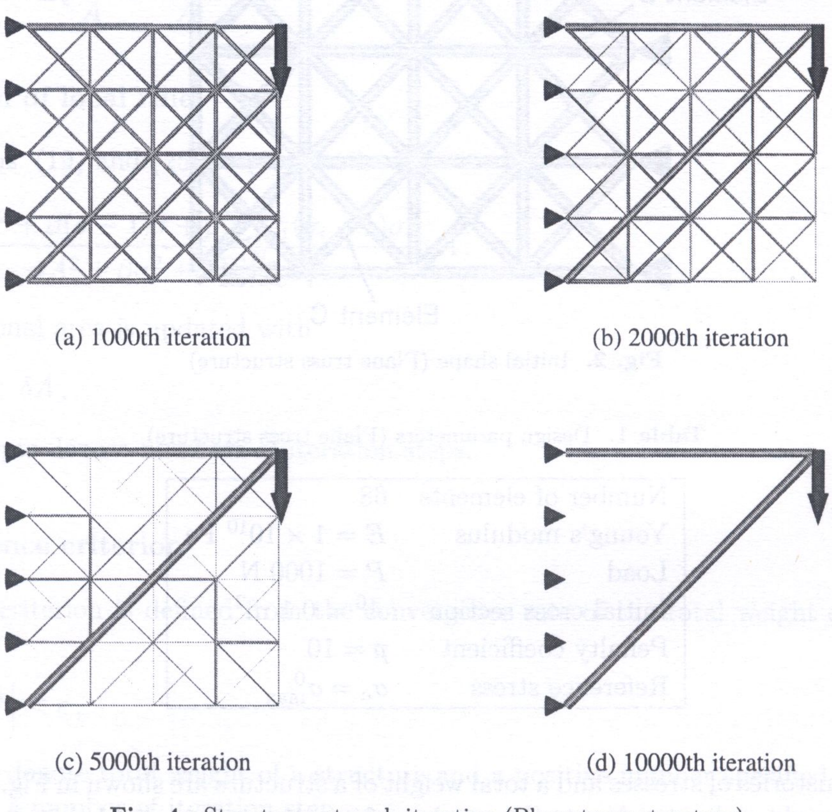


Fig. 4. Structures at each iteration (Plane truss structure)

element. The study of Bojczuk and Mróz [?] indicates that the lower chord of the optimal profile should follow a parabolic rather than a straight line. The reason why the final profile by the present method is different from their result is the initial placement of the elements. Since the elements are placed in the specific directions (horizontal, vertical and 45-degree directions), it may be difficult to obtain the parabolic line. Convergence histories of objective functions of typical elements are shown in Fig. 5. The abscissa and the ordinate denote number of iteration step and objective function defined as

$$H = \alpha H_1 + \beta H_2 \,.$$

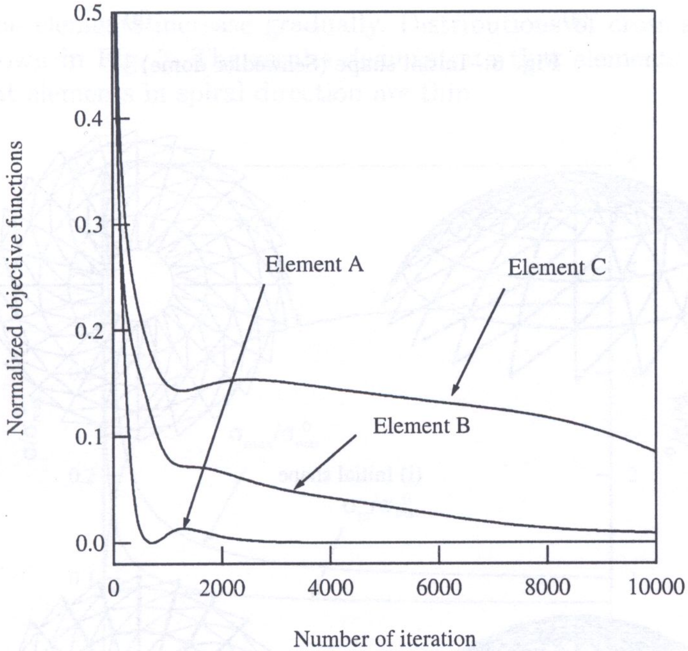


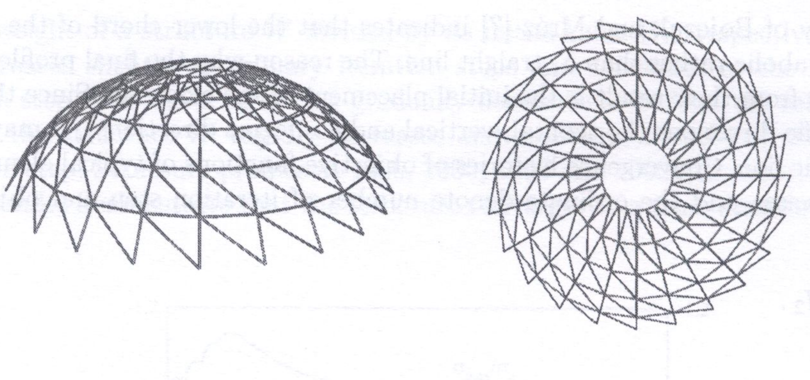
Fig. 5. Convergence histories of objective functions (Plane truss structure)

The labels A, B and C indicate the elements shown in Fig. 2. The results demonstrate that the objective functions will convergence to zero and that the convergence rate of the objective functions of the element B and C is slower than that of the element A. While the area of the element A increases, the elements B and C will finally disappear. Low convergence rate of the objective functions of the elements B and C probably mean that the disappearance of the elements is time-consuming.

4.2. Schwedler dome

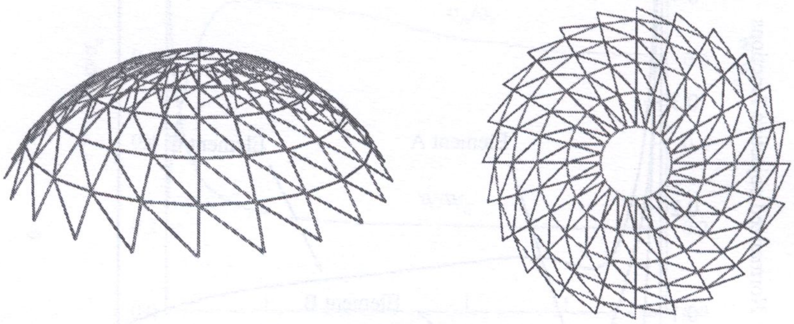
A second example is a three-dimensional truss structure "Schwedler dome" shown in Fig. 6. All nodes on ground are fixed in all directions and loads of P are applied to all nodes in vertical direction. Design parameters are shown in Table 2. Topology of the structure is fixed and cross sectional areas of each element are updated.

Convergence histories of total weight and stresses are shown in Fig. 8. The abscissa and the left- and right-ordinates denote number of iteration step, stress and total weight, respectively. The maximum and mean stresses decrease gradually and finally go into the reference stress $\sigma_c = 0.1 \times \sigma^0_{\max}$. A total weight of the structure increases gradually and finally goes into 3.7 times of an initial weight. In this case, the reference stress is specified to be very small and therefore, cross sectional

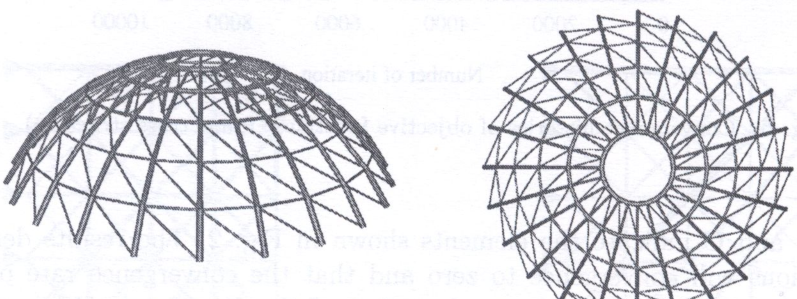


(a) (b)

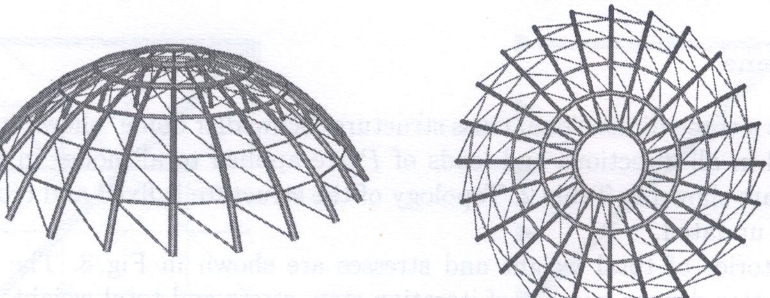
Fig. 6. Initial shape (Schwedler dome)



(i) Initial shape



(ii) 100th iteration



(iii) 1000th iteration

Fig. 7. Structures at each iteration (Schwedler dome)

Table 2. Design parameters (Schwedler dome)

Number of nodes	144
Number of elements	360
Young's modulus	$E = 2 \times 10^{11}$ Pa
Load	$P = 500$ N
Initial cross section	$A^0 = 0.1$ m^2
Penalty coefficient	$p = 5$
Reference stress	$\sigma_c = 0.1 \times \sigma^0_{\max}$

areas of almost of the elements increase gradually. Distributions of cross sectional areas at each iteration step are shown in Fig. 7. The results demonstrate that elements in radial direction are getting thick and that elements in spiral direction are thin.

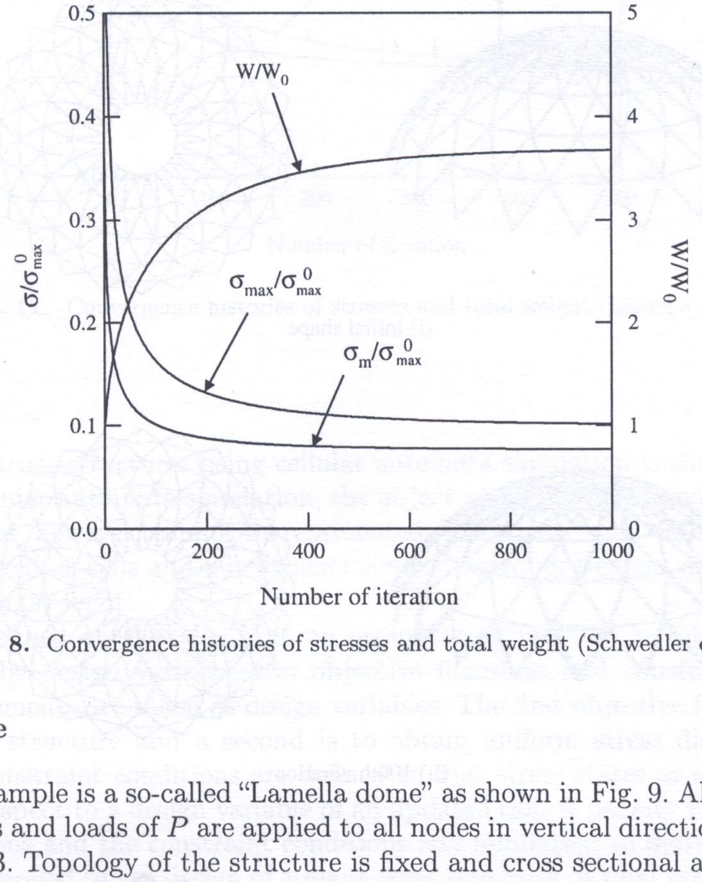


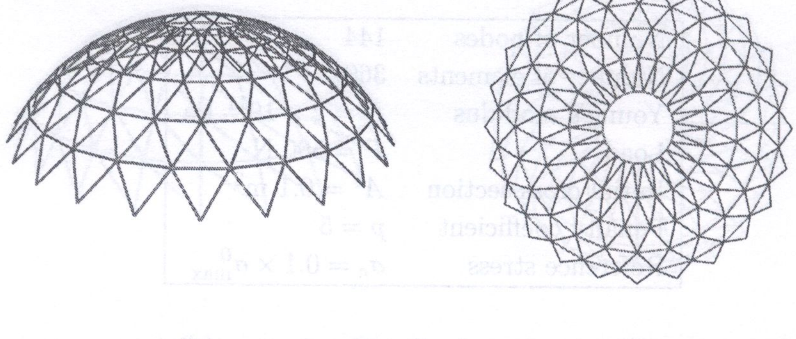
Fig. 8. Convergence histories of stresses and total weight (Schwedler dome)

4.3. Lamella dome

A final numerical example is a so-called "Lamella dome" as shown in Fig. 9. All nodes on ground are fixed in all directions and loads of P are applied to all nodes in vertical direction. Design parameters are shown in Table 3. Topology of the structure is fixed and cross sectional area of each element is updated.

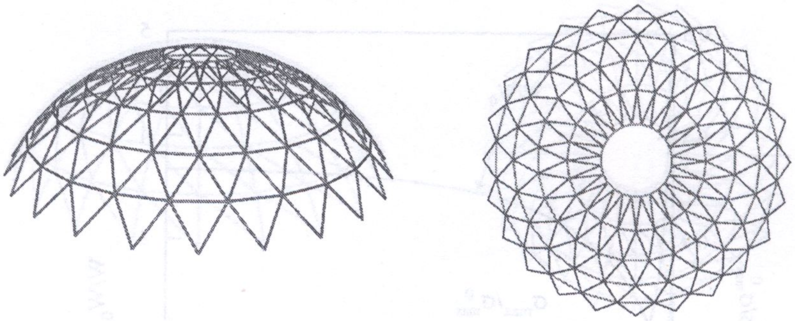
Table 3. Design parameters (Lamella dome)

Number of nodes	144
Number of elements	360
Young's modulus	$E = 2 \times 10^{11}$ Pa
Load	$P = 500$ N
Initial cross section	$A^0 = 0.1$ m^2
Penalty coefficient	$p = 5$
Reference stress	$\sigma_c = 0.1 \times \sigma^0_{\max}$

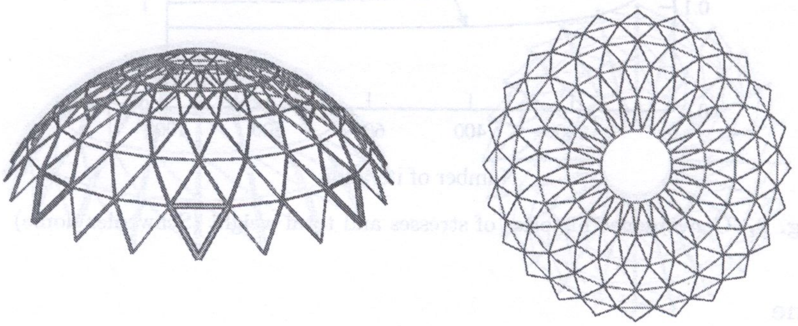


(a) (b)

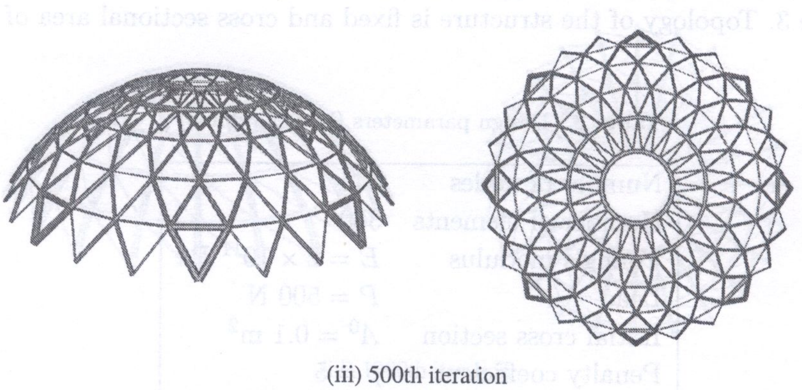
Fig. 9. Initial shape (Lamella dome)



(i) Initial shape



(ii) 100th iteration



(iii) 500th iteration

Fig. 10. Structures at each iteration (Lamella dome)

Distributions of cross sectional areas at each iteration step are shown in Fig. 10. Convergence histories of total weight and stresses are shown in Fig. 11. The results demonstrate that maximum and mean stresses converge to a reference stress and a total weight of the structure gradually increases.

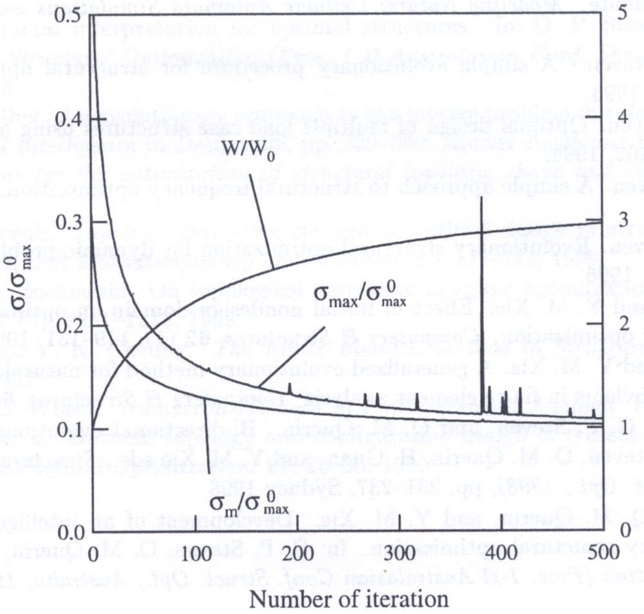


Fig. 11. Convergence histories of stresses and total weight (Lamella dome)

5. Conclusions

A design scheme of truss structures using cellular automata simulation is shown in this paper.

In traditional cellular automata simulation, the object under consideration is divided into square cells. Truss structures, however, cannot be represented with square cells. So, in the present scheme, elements are considered as cells and all elements connecting to an element are considered as neighborhood cells (elements) for it.

A local rule is defined analytically from an optimization problem as follows. An optimization problem is defined by design variable, two objective functions and constraint conditions. Cross sectional areas of elements are taken as design variables. The first objective function is to minimize a total weight of a structure and a second is to obtain uniform stress distribution on a whole structure. Special constraint conditions are defined so that stress states at each neighborhood cell are invariant with respect to a design variable of an updated cell. A penalty function is defined from the objective functions and the constraint conditions and minimized to derive a local rule.

The rule is first applied to the design of a plane truss structure. A final profile can be determined successfully but the number of iterations is relatively large. To discuss the problem, the convergence histories of the objective functions of some elements are estimated. The results demonstrate that the converge rate of disappearing elements is slow. This is probably the reason of the relatively large number of iterations. Next, the rule is applied to the design of three-dimensional truss structures "Schwedler dome" and "Lamella dome". Final profiles can be determined successfully. We can say that the results demonstrate the validity of the present scheme.

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